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RESEARCH ARTICLE

Nonlinear Approximation for Natural Convection Flow Past A Vertical Moving Plate With Nonlinear Thermal Radiation Effect

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ABSTRACT

This piece of work contains significant insight associated with the analysis of fluid transport in the vicinity of a constantly moving vertical plate with nonlinear thermal radiation. The heat transport at the wall surface is assumed to be influenced by convective boundary conditions. Furthermore, thermal transport is considered to be enhanced by nonlinear temperature variation with temperature (NDT). The boundary layer approximation equations are simplified through suitable alteration known as the similarity transformation. The resulting ODEs are translated into the IVP via shooting techniques and then integrated using the RKF45 algorithm in Maple. The impact of the dimensionless parameters dictating the fluid behaviour is demonstrated via graphs and tables. In the cause of the analysis, it is observed that the heat transfer enhances when the fluid flow in the direction +ve *x*-axis whereas the plate moves in the direction of the -ve x-axis but decreases when the plate and fluid move in the same direction. The skin friction coefficient decreases when the fluid flow is directed toward the +ve *x*-axis whereas the plate moves toward the -ve x-axis but is enhanced when the plate and the fluid move in the same orientation. The temperature and velocity profiles appreciate with the nonlinear thermal radiation when the motion of the plate and the fluid are on the same axis. The temperature gradient near the wall depreciated gradually due to nonlinear thermal radiation growth but appreciate in the free stream.

Keywords: Natural convection, Thermal radiation, Moving vertical plate.

1 Introduction

The quest to enhance the thermal transport past a boundary layer has always been the motivation behind the boundary layer flow analysis. This development has attracted the attention of several scholars across the globe in the past few years. Recent investigations revealed that boundary layer flow is applied in engineering activities such as golf ball aerodynamics, mixing capability, re-attachment heat and mass transfer, species transport etc. Earlier investigations of boundary layers such as [1] and [2] constrained their analyses to a fixed plate past a horizontal wall. The concept of moving plates is a new trend in boundary layer analysis due to its progressive application in heat and mass transfer-related analysis. [3] presented the role of heat generation/absorption, heat transfer and mass transfer and chemical reaction past a wedge. [4] inspected the dynamics features of flow in the boundary layer with thermal radiation. They established that a thinner boundary layer is observed at higher values of the Prandtl number. When examining the role of convective boundary conditions on the flow past a horizontal plate, [5] emphasised that the convective parameter is associated with the decline in heat transfer. [6] studied the role of CNTs on an MHD Casson Marangoni boundary layer flow past a porous medium with combined effects of thermal radiation and suction/injection. [7] reported a unique design of inverse multiquadric radial basis neural network to study MHD nanofluid boundary layer flow over a wedge submerged in a porous medium with viscous and radiation effects. Other relevant literature can be seen in [8]–[11].

The viscosity of the fluid in the vicinity of the boundary layer is a significant phenomenon to investigate as it enhances fluid transport. It is generally influenced either by external mechanisms (such as heat source/sink, thermal radiation, suction/injection etc.) or internal force (due to buoyancy). It is paramount to stress that at considerably high temperatures, the thermal radiation effect can influence temperature distribution and heat transfer. In addition, at high operating temperatures, thermal radiation is useful in combustion, fire science and industrial design. The knowledge of thermal radiation is useful in nuclear plants, power generation systems, gas turbines, liquid metal fluids etc. [12] explored the implication of the three-dimensional flow of an eying-Powell fluid with an induced magnetic field. Analytic solution for natural convection flow through a porous medium with combined effects of viscous dissipation and thermal radiation has been reported in [13]. An analysis of NDT on natural convection flow past a vertical place with an emphasis on the thermal radiation effect has been reported in [14]. In the report presented by [15] on the collective importance of suction/injection and thermal radiation on MHD natural convection flow between two vertical porous plates, they established that the temperature profile is proportional to the Prandtl number. [16] presented the importance of thermal radiation and heat generation on the flow of a magnetic Eyring-Powell hybrid nanofluid in a porous medium. In a study conducted by [17] on the significance of thermal radiation on thermostatically stratified MHD fluid flow through an accelerated vertical porous plate with viscous dissipation, the concluded that as the radiation parameter increases, the temperature profile decreases. Furthermore, [17]–[19] demonstrated the implications of thermal radiation on different geometries. [20] considered the importance of thermal conductivity and radiation past a stretching plate with convective boundary conditions. Some relevant articles can be found in [21]–[24].

Recent investigations of flow properties in the boundary layer considered the dynamic interaction between the constantly moving boundary layer and the fluid flow. This phenomenon has captured the attention of various investigators resulting in the revisit of some literature analysed using the fixed plate. The significance of the constantly moving plate is the ability to accurately predict the heat transfer and shear stress at the plate surface and the ability to obtain the dual (upper and lower branch) solution to dimensionless equations. However, several researchers reported that the upper branch solution is consistent and has more engineering importance compared to the lower branch solution. To date, the consistency of the lower branch solution is yet to be proved by any worker. [25] numerically examined the thermal process past moving flat plates. The report presented by [26] gave a similarity solution for natural convection flow and mass transfer past a moving plate. [27] analysed non-linear natural convection flow past a constant moving wall.

From the aforementioned articles, the impact of thermal radiation on boundary layer flow has been explored by various workers. However, most analyses considered linear thermal radiation rather than nonlinear thermal radiation, fixed plate rather than a constant moving plate, linear density variation with temperature (LDT) rather than nonlinear density variation with temperature (NDT) and constant surface boundary condition rather than convective boundary condition. In the present paper, we studied the natural convection boundary layer flow near a moving vertical wall influenced by nonlinear thermal radiation having convective boundary conditions. The role of NDT in the buoyancy term is also taken into an account.

2 Governing equations

Consider the case of free convection flow of incompressible viscous fluid flows past a constant moving vertical plate under the influence of nonlinear thermal radiation. Supposed that the velocity of the plate and the fluid are respectively U_w and U_∞. Utilizing the Boussinesq approximation, the mathematical model governing the boundary layer fluid transport can be expressed as

$$\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\left[\beta_0\left(T-T_\infty\right)+\beta_1\left(T-T_\infty\right)^2\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left[\frac{\partial^2 T}{\partial y^2}-\frac{1}{k}\frac{\partial q_r}{\partial y}\right], \tag{3}$$

where T is the dimensional temperature, q_r denote the radiative heat flux, β_0 and β_1 are linear and nonlinear thermal expansion coefficient, α the thermal diffusivity, ν the kinematic viscosity and k the thermal conductivity. Here, the viscous dissipation is considered to be very negligible since the flow is laminar. According to [13], [28], [29] the radiative heat flux q_r can be defined via Roseland diffusion approximation as

$$q_r=-\frac{4\sigma\partial T^4}{3k^*\partial y}. \tag{4}$$

Here, σ represents the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. It is significant to establish that the Rosseland approximation is applied only in an optical thick fluid. Nevertheless, it has been widely used in the analysis such as the effect of radiation via gases at low density, nuclear explosion etc. Through convection, the constantly moving wall is heated by a hot fluid at a temperature T_f that produces a resultant heat transfer coefficient h_f. Thus, the thermal boundary conditions relevant to this study are defined as

$$\begin{aligned}&u(x,0)=U_w,\quad v(x,0)=0\\&u(x,\infty)=U_\infty\end{aligned} \tag{5}$$

$$\begin{aligned}&-k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T_\infty(x,0)\right],\\&T(x,\infty)=T_\infty.\end{aligned} \tag{6}$$

The dimensional and similarity variables are defined as

$$\begin{aligned}&\eta=y\sqrt{\frac{U_\infty}{\nu x}},\quad u=U_\infty f'(\eta),\quad C_T=\frac{T_\infty}{T_f-T_\infty},\quad R=\frac{4\sigma\left(T_f-T_\infty\right)^3}{k^*k},\\&v=\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}\left(\eta f'(\eta)-f\right),\ \theta=\frac{T-T_\infty}{T_f-T_\infty}\quad \Pr=\frac{\mu C_p}{k},\ \delta=\frac{\beta_1\left(T_f-T_\infty\right)}{\beta_0}\end{aligned} \tag{7}$$

The reference velocity U is defined as the sum of the plate velocity and fluid velocity and is expressed as

$$U=U_\infty+U_w \tag{8}$$

Incorporating the dimensional variables into the governing equations, the dimensionless equations reduce to

$$f'''(\eta)+\frac{1}{2}f(\eta)f''(\eta)+Gr_x\left[\theta(\eta)+\delta\theta^2\right]=0 \tag{9}$$

$$\theta''(\eta)[1+\frac{4R}{3}(\theta+C_T)^3]+4R[C_T+\theta]^2[\theta'(\eta)]^2+\frac{1}{2}\Pr\theta'(\eta)f(\eta)=0 \tag{10}$$

$$f(0)=0,\quad f'(0)=\lambda,\quad f'(\eta\to\infty)=1-\lambda \tag{11}$$

$$\theta'(0)=-Bi_x\left[1-\theta(0)\right],\ \theta(\eta\to\infty)=0 \tag{12}$$

The convective parameter Bi_x, Grashof number Gr_x and the velocity ratio λ are defined as

$$Bi_x = \frac{h_f}{k}\sqrt{\frac{vx}{U_\infty}} \tag{13}$$

$$Gr_x = \frac{\nu xg\left(T_f - T_\infty\right)}{U^2{}_\infty} \tag{14}$$

$$\lambda = U_\infty / (U_\infty + U_w), \qquad U_\infty + U_w \neq 0 \tag{15}$$

Several workers such as [5] and [30] established that for the governing equations to have a similarity solution, Bi_x and Gr_x should be constant not as expressed in Eqns (13 and 14). For the three quantities to be constants, h_f and β_0, β_1 must be respectively directly proportional to $x^{-1/2}$ and x^{-1}. Therefore, with n and m as the constants of proportionality, we write

$$\beta_0 = mx^{-1}, \; \beta_1 = nx^{-1}, \qquad h_f = cx^{-1/2} \tag{16}$$

Substituting Eqn (16) in Eqns(13) and (14) yield

$$Bi_x = \frac{c}{k}\sqrt{\frac{v}{U_\infty}}, \qquad Gr_x = \frac{\nu mg\left(T_f - T_\infty\right)}{U^2{}_\infty} \tag{17}$$

This communication reduces to the examination conducted by [31] when $\lambda = 0, \; Gr_x = 0$, $\delta = 0$ and to [5] when $\lambda = 0, \; Gr_x = 0, \; R = 0, \; \delta = 0$. Three possible interactions between the plate and fluid are defined;

Case 1 (the same direction): The case $0 < \lambda < 1$ corresponds to when the fluid and plate move in the same orientation.

Case 2 (opposite direction): The case $\lambda < 0$ corresponds to when the fluid flow in the free stream region is directed toward the +ve *x*-axis while the plate moves toward the -ve x-axis.

Case 3 (opposite direction): The case $\lambda > 1$ corresponds to when the fluid flow is directed toward the -ve x-axis while the plate moves toward the +ve x-axis.

Additionally, the cases $\lambda = 1$ and $\lambda = 0$ respectively represent a constant moving plate with a fixed free stream and a fixed plate with constant variation in the free stream which has already been considered by several workers such as [32] and [4]. The present communication considered cases 1 and 2 only.

2.1 Numerical Solution through Shooting Method

The reduced dimensionless equations and BCs are reduced to the initial value problem through the shooting method. We let

$$\begin{aligned} f' &= y_1, \\ y_1' &= y_2, \\ y_2' &= -\frac{1}{2} f\, y_2 - Gr_x\left[\theta + \delta\theta^2\right] \end{aligned} \tag{18}$$

$$\theta' = y_3$$
$$y_3' = \frac{-1}{[1+\frac{4R}{3}(\theta + C_T)^3]}\left[4R[C_T+\theta]^2[y_3(\eta)]^2 + \frac{1}{2}\Pr y_3(\eta) f(\eta)\right] \tag{19}$$

With the initial conditions

$$\begin{aligned} &f(0)=0, \quad y_1(0)=\lambda, \quad y_2(0)=\alpha_1 \\ &y_3(0)=-Bi_x\left[1-\theta(0)\right], \ \theta(0)=\alpha_2 \end{aligned} \tag{20}$$

To integrate the above IVP, we need the values of $y_2(0)$ and $\theta(0)$ however, such values are not given in the BCs. Suitable values of $\theta(0)$ and $y_2(0)$ e.g α_1 and α_2 are used to integrate Eqns (19) and (20). The computed results for $y_2(0)$ and $\theta(0)$ is compared with $f'(\infty)=1-\lambda$ and $\theta(\infty)=0$. The numerical values of α_1 and α_2 are adjusted till an appropriate approximation is found. Lastly, the resulting IVP along with the initial conditions is solved in Maple software using the RKF45 method.

3 Results and Discussion

The flow behaviour in the boundary layer predisposed by nonlinear thermal radiation is examined. The plate is considered to be in constant motion with convective surface boundary conditions. The mathematical model comprised nonlinear density variation with temperature (NDT) in the buoyancy term. The nonlinear and couple equation is simplified through a suitable transformation known as similarity transformation. The reduced boundary value problem is translated to the IVP through a shooting method and finally solved through RKF45 in Maple. The importance of the prominent parameter present in the mathematical model influencing fluid transport is examined. The range of values of the dimensionless parameters used during computation is $-0.3 \le \lambda \le 0.8$, $0.5 \le R \le 10$, $0.1 \le Gr_x \le 1.0$, $0.1 \le Bi_x \le 2.0$, $0.1 \le \delta \le 2.0$, $\Pr = 0.72$ and $C_T = 0.2$.

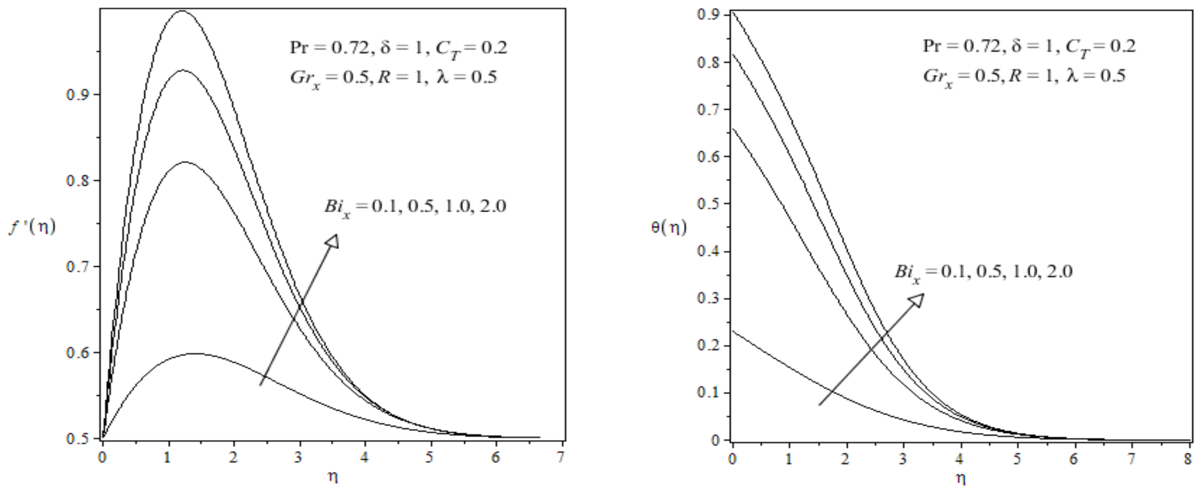


Figure 1: *Flow formation and temperature distribution curves various values of* Bi_x

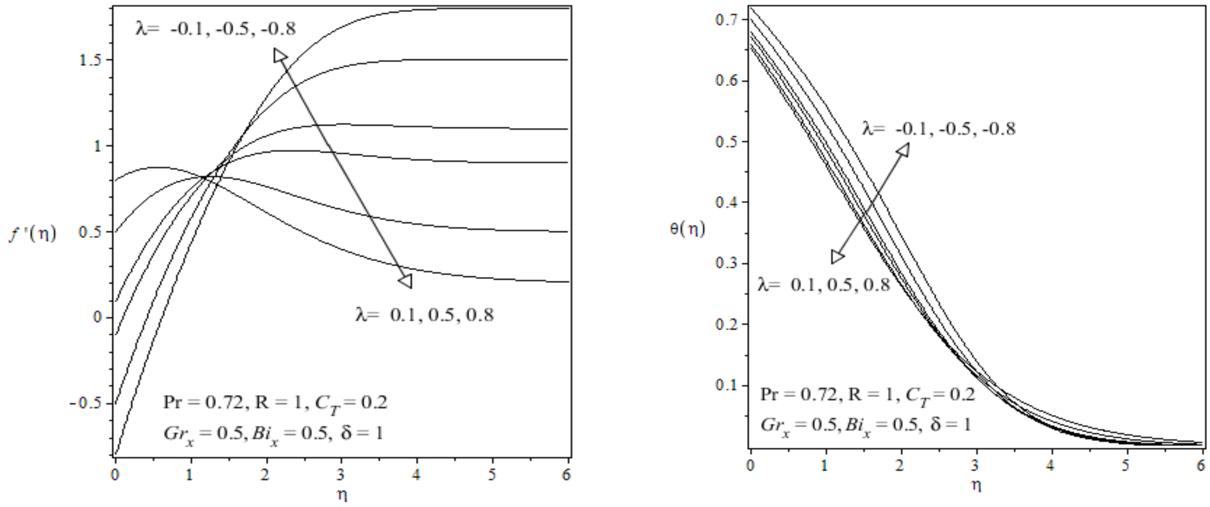


Figure 2: *Flow formation and temperature distribution curves various values of* λ

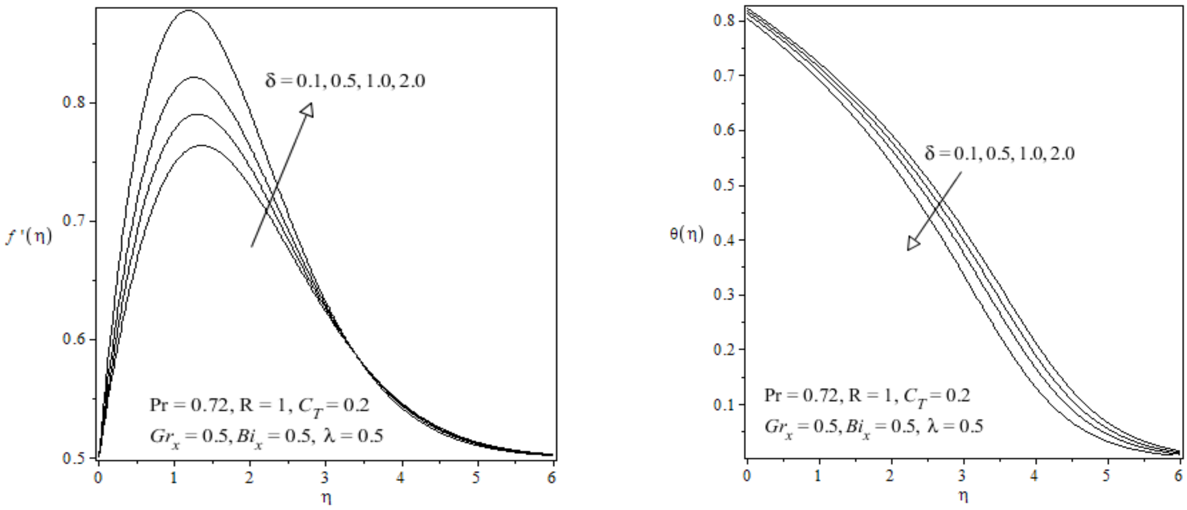
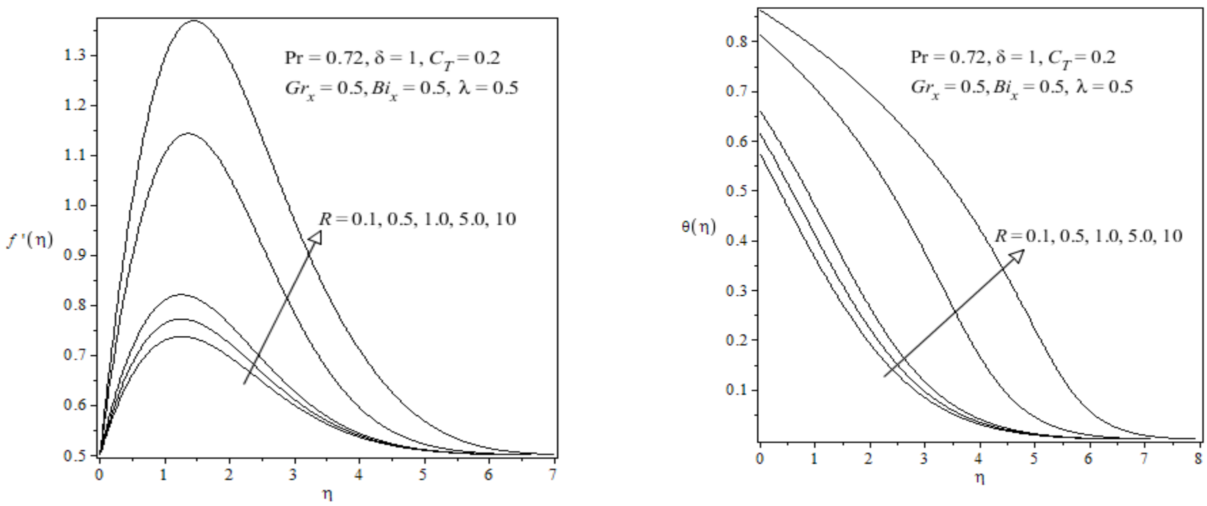


Figure 3: *Flow formation and temperature distribution curves various values of* δ



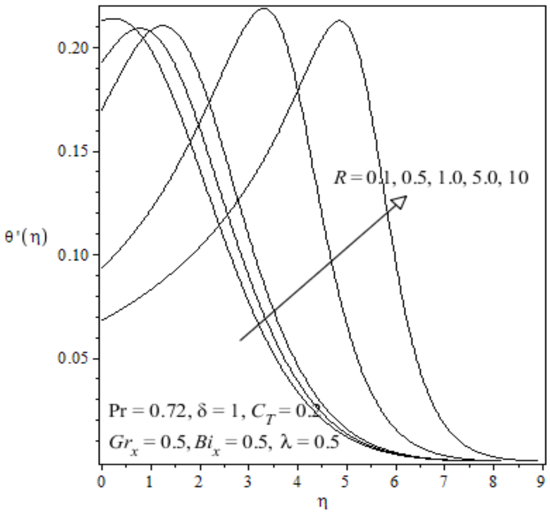


Figure 4: *Flow formation temperature and temperature gradient distribution curves various values of* R.

The implication of the convective heat transfer parameter (Bi_x) on the velocity and temperature curves for case 1 is demonstrated in Figure 1. The figure reveals that when the plate and the fluid move in the same direction, increasing Bi_x lead to a significant increase in the flow formation and temperature curve. Figure 2 depicts the relevance of the velocity ratio parameter (λ) on the velocity and temperature distribution curves for cases 1 and 2. It is remarkable to stress that the temperature distribution curve is enhanced when the fluid flow is directed toward the +ve *x*-axis whereas the plate moves toward the -ve x-axis (case 2). However, contrary behaviour is seen when the fluid and the plate motion are in the same direction (case 1). Furthermore, the velocity profile exhibits the same behaviour as that of the temperature in the free stream but the contrary is true near the plate. An intersection between the two phenomena is recorded at $\eta \approx 1$. The importance of the nonlinear parameter (δ) for case 1 on the flow formation and temperature profile is described in Figure 3 where the flow formation exponentially grows as δ propagate. The temperature distribution on the other hand drops with the higher values of δ. The importance of the nonlinear thermal radiation parameter (R) for case 1 is demonstrated in Figure 4. It is remarkable to state that the temperature and velocity profiles appreciate with the intensity of *R* when the motion of the plate and the fluid are in the same orientation. The shape growth in the flow formation profile due to R augment is associated with the fact that the radiative heat flux applied at the plate surface gives raised to the temperature distribution and consequently increases the velocity profile. The temperature gradient near the wall depreciated gradually due to R growth but appreciate in the free stream.

Table 1: *Correlation of the current analysis with [30] for* $R=\delta=\lambda=0$.

			[30]			Present Work	
Pr	Gr_x	Bi_x	$-\theta'(0)$	$f''(0)$		$-\theta'(0)$	$f''(0)$
0.72	0.1	0.1	0.07507	0.36881		0.07507	0.36881
0.72	0.1	1.0	0.23750	0.44036		0.23750	0.44036
0.72	0.1	10	0.30559	0.46792		0.30559	0.46792
0.72	0.5	0.1	0.07613	0.49702		0.07613	0.49702
0.72	1.0	0.1	0.07704	0.63200		0.07704	0.63200
3.00	0.1	0.1	0.08304	0.34939		0.08304	0.34939
7.10	0.1	0.1	0.08672	0.34270		0.08672	0.34270

Table 2: *Correlation of the current analysis with some related literature.*

λ	[1]	[33]	[34]	[35]	[36]	[24]	Present study
0	0.332	0.33206		0.33206	0.3321	0.332060	0.33206
0.5						0	0
1.0			-0.44375		-0.4438	-0.443751	-0.443751

Table 3: *The impact of the prominent parameters on the Nusselt number* $-\theta'(0)$ *for* $C_T = 0.2,\ \delta = 1$ *and* $\Pr = 0.72$.

			Case 2				Case 1		
Bi_x	Gr_x	R	$\lambda = -0.1$	$\lambda = -0.2$	$\lambda = -0.3$		$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.8$
0.1	0.5	1.0	0.074437	0.073772	0.073001		0.075524	0.077025	0.077741
0.5	0.5	1.0	0.160137	0.157947	0.155581		0.164038	0.170143	0.173402
1.0	0.5	1.0	0.174406	0.172360	0.170141		0.178022	0.183523	0.186271
0.5	0.1	1.0	0.139187	0.135805	0.131934		0.144782	0.152357	0.155361
0.5	0.4	1.0	0.156264	0.153919	0.151368		0.160400	0.166739	0.169994
0.5	0.8	1.0	0.169303	0.167415	0.165403		0.172732	0.178331	0.181548
0.5	0.5	0.1	0.139187	0.135805	0.131934		0.144782	0.152357	0.155361
0.5	0.5	0.5	0.160137	0.157947	0.155581		0.164038	0.170143	0.173402
0.5	0.5	0.8	0.169303	0.167415	0.162716		0.172732	0.178331	0.181548

Table 4: *The impact of the prominent quantities on skin friction* $f''(0)$ *for* $C_T = 0.2,\ \delta = 1$ *and* $\Pr = 0.72$.

			Case 2				Case 1		
Bi_x	Gr_x	R	$\lambda = -0.1$	$\lambda = -0.2$	$\lambda = -0.3$		$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.8$
0.1	0.5	1.0	0.597647	0.653758	0.703726		0.469015	0.157669	-0.112284
0.5	0.5	1.0	1.070163	1.141546	1.208625		0.915629	0.566649	0.277609
1.0	0.5	1.0	1.278458	1.351243	1.419836		1.121427	0.768500	0.477228
0.5	0.1	1.0	0.560492	0.610422	0.652478		0.440849	0.140243	-0.124667
0.5	0.4	1.0	0.960010	1.027834	1.090951		0.811595	0.471890	0.188005
0.5	0.8	1.0	1.364320	1.443783	1.519692		1.195533	0.823932	0.521607
0.5	0.5	0.1	0.560492	0.610422	0.652478		0.440849	0.140243	-0.124667
0.5	0.5	0.5	1.070163	1.141546	1.208625		0.915629	0.566649	0.277609
0.5	0.5	0.8	1.364320	1.443783	1.519692		1.195533	0.823932	0.521607

Some physical measures of industrial and scientific relevance considered in this communication are the Nusselt number $-\theta'(0)$ and skin friction $f''(0)$. Table 1 is a correlation between the current result with the theoretical analysis of thermal radiation and buoyancy effects near a fixed vertical plate presented by [14] and [30] respectively. Comparing Table 1 revealed that the presence of density variation with temperature in the buoyancy term enhanced the heat transfer rate and wall shear stress. This shows that in a physical situation/industrial activity where the rate of heat transfer needs to be optimised for optimal performance, nonlinear thermal radiation and temperature-dependent density variation could be used.

Table 2 also demonstrates the correctness of the present analysis when the plate is considered to be fixed and horizontal with some related literature. It is obvious from the tables that the current results obtained are promising. The implication of the embedded parameters on the Nusselt number for cases 1 and 2 with δ effect is portrayed in Table 3. The heat transfer rate drops with Bi_x, Gr_x and R increases for the two cases of the plate and fluid motion under consideration. However, the heat transfer rate increases when the fluid flow is directed toward the +ve *x*-axis whereas the plate moves toward the -ve x-axis but decreases when the fluid and the plate move in the same orientation. Regarding the skin friction, an observable increase is seen in the skin friction coefficient when the fluid and the plate move in the same orientation whereas a significant decrease is recorded when the fluid flow is directed toward the +ve x-axis while the plate moves toward the -ve x-axis whereas decreases when the fluid and the plate moves in the same orientation as shown in Table 4. Furthermore, the numerical values for skin friction coefficient propagate with Bi_x, Gr_x and R augment for the two cases of fluid and plate motion under examination.

4 Conclusion

The theoretical investigation of the flow behaviour near a vertical moving plate influenced by nonlinear thermal radiation with the thermal boundary condition of the third kind is presented through a numerical approach. The influence of the embedded parameter dictating fluid transport is studied. Our final remark includes; the heat transfer rate enhances when the fluid flow is directed toward the +ve x-axis while the plate moves toward the -ve x-axis but decreases when the plate and the fluid move in the same orientation. The heat transfer decrease with Bi_x, Gr_x and R propagation. The skin friction coefficient decreases when the fluid flow is directed toward the +ve x-axis while the plate moves toward the -ve x-axis but is enhanced when the plate and the fluid move in the same orientation. The skin friction coefficient is enhanced with Bi_x, Gr_x and R augment. The temperature and velocity profile appreciate with R when the motion of the plate and the fluid are in the same orientation. The temperature gradient near the wall depreciated gradually due to R growth but appreciate in the free stream.

5 Declaration

5.1 Competing Interests

The authors declare that there is no conflict of interest exist.

5.2 Publisher's Note

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