RESEARCH ARTICLE

Unconstrained and Constrained Predictive Control for the Multivariable Process with Non-minimum Phase

Zidane Zohra

Team of Applied Physics and New Technologies, Department of Physic, Polydisciplinary Faculty, University of Sultan Moulay Slimane, Morocco

* Corresponding author email: zidane.zohra@gmail.com

ABSTRACT

Non-minimum phase Multi-input Multi-Ouput (MIMO) systems are known to be difficult to control. Model Predictive Control (MPC) algorithms are powerful control design methods widely applied to industrial processes. The handling of various input constraints in the MPC problem of ARIMAX non-minimum phase MIMO systems is considered. This approach is applied for control of industrial quadruple tanks. However, there is no easy way to solve the problem of constraints. The methods based on the quadratic programming (QP) technique are used to solve the constrained optimization problem. A comparative study of unconstrained and constrained control system behavior is given. Some illustrative simulation results for a considered system are presented and discussed. Encouraging results are obtained that motivate for further investigations.

Keywords: ARIMAX systems, MIMO systems, Model Predictive Control, Non-minimum Phase systems.

1 Introduction

Developing efficient control strategies for the control of multivariable system for many areas of engineering is quite challenging due to cost and time consuming on model identification. It is necessary for the controller to have a prototype of the real process to get knowledge about the process it should control. Most of the difficult problems in industries can be preferred to be solved by using Model Predictive Controller as it has so much impact on industrial control [1].

Model Predictive Control (MPC) seems to be one of the most popular topics in academic research and process control engineering mainly because of its simplicity and successful industrial applications [2]-[3]. MPC is the most widely used controller at present due to its ability to handle multivariable process and constraints in a simple way. Constraints of different kind are ubiquitous in control engineering applications; the way of handling them in control system design is an important question. However, this does not often happen in the design of control algorithms reported in the literature. Disregarding constraints or imposing them on the control signal in a heuristic way can cause performance deterioration or even instability, especially in predictive control of unstable systems. Design formulation, ability to handle constraints, online process optimization and simplicity of the design are the major aspects of model predictive control that make it attractive to practitioners and researchers [4].

Taking constraints into account in the design stage leads inherently to a solution of constrained optimization problem. It is well-known that quadratic programming (QP) technique can be applied to solve miscellaneous types of predictive control problems under constraints [5]. In this paper, The MPC is considered which perhaps one is of the most successful representative amongst predictive control proposals because it is capable to integrate optimal control, dead time and unstable processes control, multivariable

control, non-minimum phase systems and future references when available [1]. The MPC is not a specific control strategy but an ample range of control methods where the control signal is obtained by minimizing an objective function [6].

MPC algorithms usually assume that all signals have an unlimited range, although real processes have constraints – limited range of action, limited action increment, constrained output, etc. For this reason, it is necessary to use MPC controllers to cope with constrained inputs (amplitude and increment). The application of the QP to solve the MPC under constraints is widely used; see for example the comments given in [7] and [8].

2   MPC control algorithm

Predictive control philosophy, aiming is creating an anticipative effect using the explicit knowledge of the trajectory in the future [1]. The MPC uses a system model to predict the future states of the system and generates a control vector that minimizes a certain cost function over the prediction horizon in the presence of disturbances and constraints. Only the first value of the computed control vector at any sampling instant is applied to the system input, and the remainder is discarded. The entire process is repeated in the next time instant. The cost function can take the form of tracking error, control effort, energy cost, demand cost, power consumption, or a combination of these factors. Constraints can be placed on the rate and range limits of the actuators and the manipulated and controlled variables (e.g., upper and lower limits of the zone temperature, supply airflow rate limits, and range and speed limits for damper positioning) [9]. A discrete cost function of MPC is given as follows [4]:

$$J = \left(y_{k+1/L} - y^*{}_{k+1/L}\right)^T Q \left(y_{k+1/L} - y^*{}_{k+1/L}\right) + u_{k/L}{}^T P u_{k/L} + \Delta u_{k/L}{}^T R \Delta u_{k/L} \quad (1)$$

Where

$y^*{}_{k+1/L}$ is a vector of future setpoint,

$y_{k+1/L}$ is a vector of future outputs,

$\Delta u_{k/L}$ is a vector of future input changes,

$u_{k/L}$ is a vector of future input.

Q output error weight matrix, R and P control weight matrix.

The constrained optimal control is specified as follow [4]:

$$u_{k/L}{}^* = \arg\min J(u_{k/L}) \quad (2)$$

The goal of the MPC is the output $y_{k+1/L}$ to follow the reference signal $y^*{}_{k+1/L}$ taking into account the control effort [4]. All physical systems have constraints. In MPC one normally defines these constraints to minimize inequalities.

Constraints in the inputs:

$$u_{k/L}^{min} \leq u_{k/L} \leq u_{k/L}^{max} \quad (3)$$

$$\Delta u_{k/L}^{min} \leq \Delta u_{k/L} \leq \Delta u_{k/L}^{max} \quad (4)$$

Constraints in the outputs:

$$y_{min} \leq y_{k+1/L} \leq y_{max} \quad (5)$$

The solution to constraint problem can be solved by the quadratic programming. The control objective criterion and prediction model are given as,

$$J = \left(y_{k+1/L} - y^*{}_{k+1/L}\right)^T Q \left(y_{k+1/L} - y^*{}_{k+1/L}\right) + \Delta u_{k/L}{}^T R \Delta u_{k/L} \quad (6)$$

Prediction model,

$$y_{k+1/L} = F_L + G_L u_{k/L} \tag{7}$$

By substituting equation (7) into (6) gives,

$$J = \Delta u_{k/L}^T H \Delta u_{k/L} + 2f^T \Delta u_{k/L} + J_0 \tag{8}$$

Where

$$H = G_L^T Q G_L + R$$
$$f = G_L^T Q (F_L - y^*_{k+1/L})$$
$$J_0 = (F_L - y^*_{k+1/L})^T$$

And the control single $u_{k/L}$ can be computed as $u_{k/L} = \Delta u_{k/L} + u_{k-1/L}$.

3   Simulation Analysis

In this paper, the Model based Predictive Controller (MPC) for a quadruple tank system is given to control the level of tank based on two inputs (u1 and u2) and two outputs (y1 and y2) MIMO system. The constrained and unconstrained MPC with integral action algorithm are implemented in the linearized model of the four-tank non-minimum phase process [10]-[11]. The constraints are handled using MATLAB "quadprog" function. The linearized discrete four-tank plant model is as follows:

$$\begin{aligned} x_{k+1} &= A x_k + B u_k \\ y_k &= D x_k \end{aligned} \tag{9}$$

Where

$$A = \begin{bmatrix} 0.9984 & 0 & 0.0026 & 0 \\ 0 & 0.9989 & 0 & 0.0018 \\ 0 & 0 & 0.9974 & 0 \\ 0 & 0 & 0 & 0.9982 \end{bmatrix}; \quad B = \begin{bmatrix} 0.0048 & 0 \\ 0 & 0.0035 \\ 0 & 0.0077 \\ 0.0056 & 0 \end{bmatrix}; D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

3.1   Unconstrained MPC Control

Implementation of unconstrained MPC is divided into simulation of non-minimum phase process and the results are plotted. The performance of the unconstrained MPC controller is evaluated using following parameters:

Appropriate values of the weighting matrices Q and R are assigned to weighted the output and input variables respectively, $Q = \begin{bmatrix} 150 & 0 \\ 0 & 150 \end{bmatrix}$, and $R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$.

[9.5 10.5] [cm] the initial level in tanks 1 and 2

The reference is chosen as a square wave.

The simulation results are obtained by using Matlab Toolbox. The simulation results of unconstrained MPC controller for the non-minimum phase system using "quadprog" are given in Figures 1 and 2, it can be observed that the MPC control without constraints produces the response with a undershoots and overshoots in both the tanks 1 and 2.
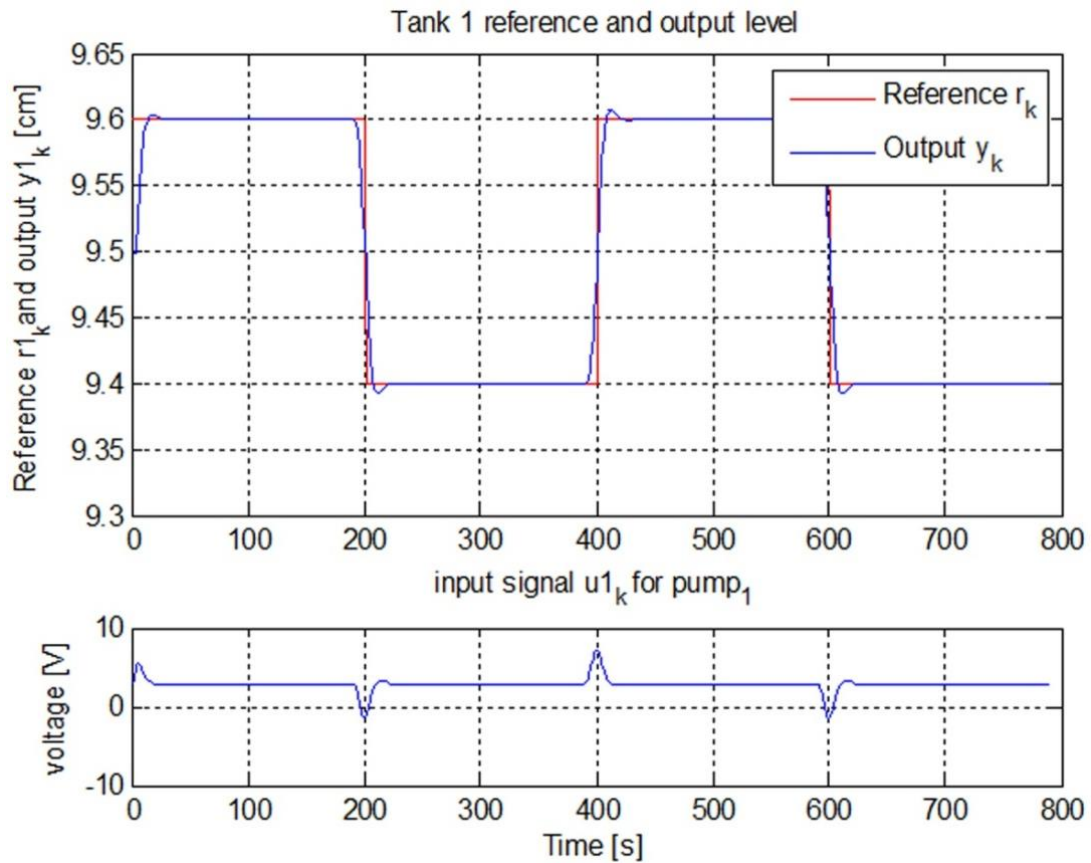
Figure 1: *Unconstrained MPC controller response for tank 1. The upper two plots illustrated the reference signal and output levels for tank 1. The lower plot is the controller input signal for pumps 1.*
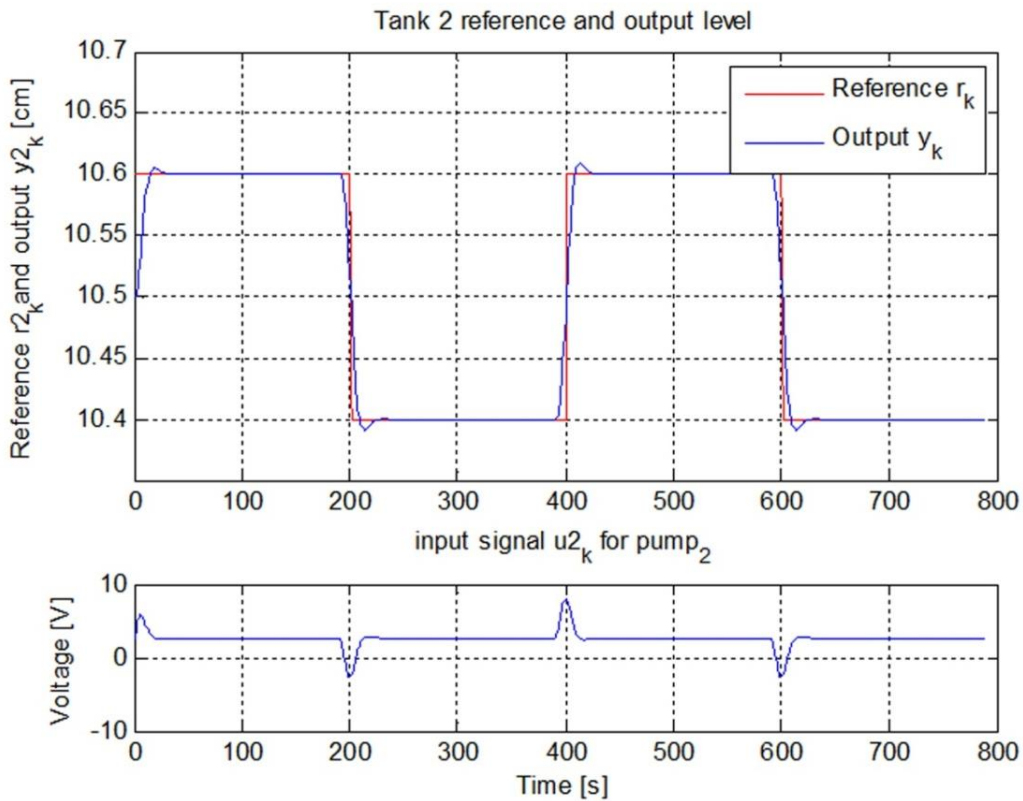

Figure 2: *Unconstrained MPC controller response for tank 2. The upper two plots illustrated the reference signal and output levels for tank 2. The lower plot is the controller input signal for pumps 2.*

3.2   Constrained MPC Control

The major objective of MPC in industrial process application is its ability to handle constraints [1]. Here the constraints are provided on both the input voltages to the pumps at the amplitude constraints $0 \leq u \leq 0.5$ and the parameter values used in the above method (unconstrained MPC control) kept the same and the simulation results are shown in figures 3 and 4. From the responses it is clearly shown that the output variables are able to track the set points given with a small undershoots and overshoots in both the tanks 1 and 2.
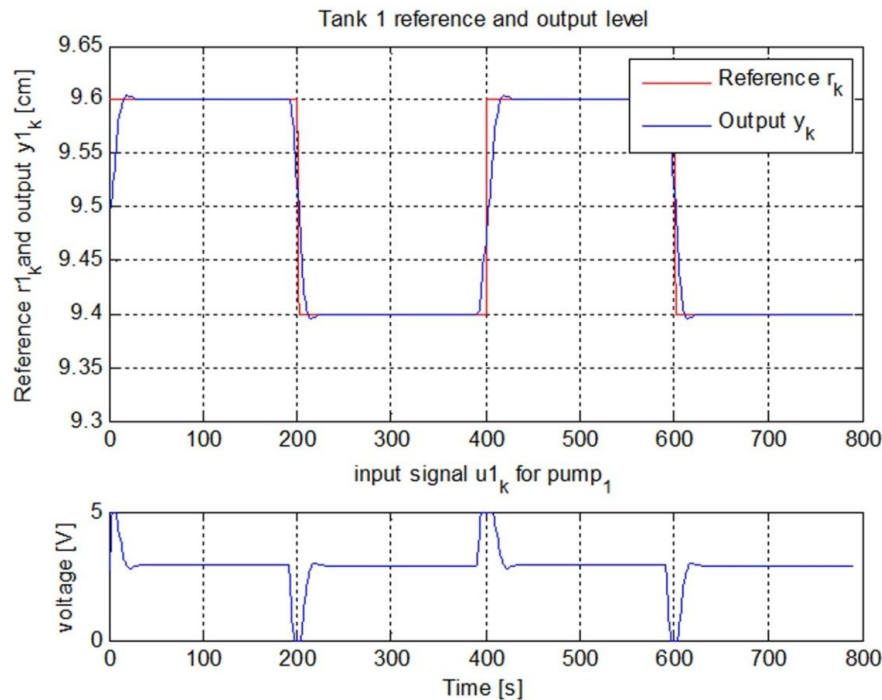

Figure 3: *Constrained MPC controller response for tank 1. The upper two plots illustrated the reference signal and output levels for tank 1. The lower plot is the controller input signal for pumps 1.*
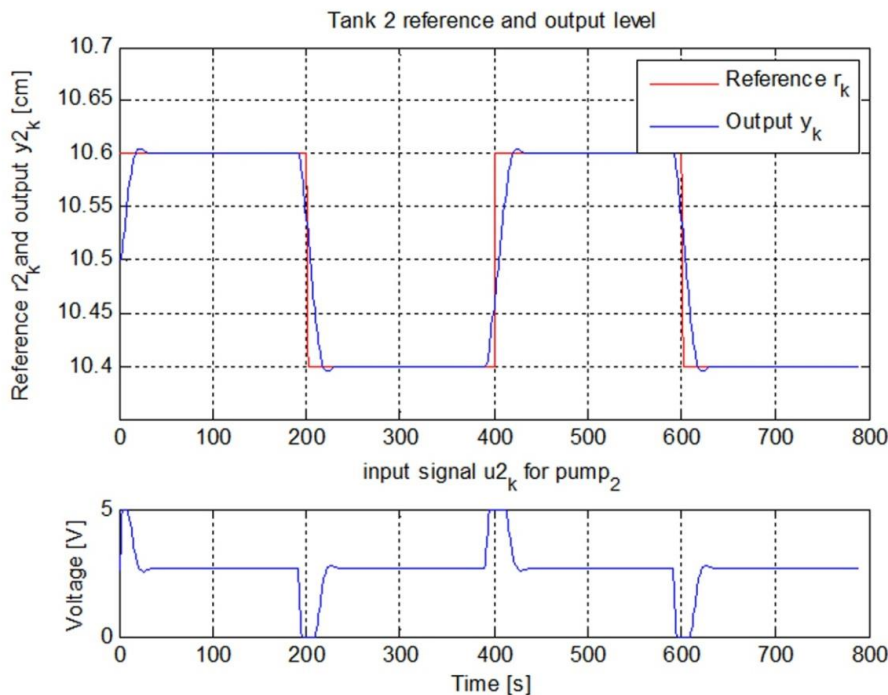

Figure 4: *Constrained MPC controller response for tank 2. The upper two plots illustrated the reference signal and output levels for tank 2. The lower plot is the controller input signal for pumps 2.*

3.3   Comparison Results

In the above figures, it can be observed the comparative results between unconstrained and constrained MPC. The output levels for tanks 1 and 2, and the controller input signal for pumps 1 and 2, under unconstrained are shown, respectively, in fig.1 and fig.2. The simulation results from implementing MPC under constrained are illustrated in fig.3 and fig.4. Best performance is characterized by best tracking, robustness, lower or no over/undershoots less or no oscillations. Based on this, it can be observed that the constrained MPC produces the best response in terms of tracking, and overshoot, cancellation of oscillation.

4   Conclusions

The Model Predictive Controller (MPC) is very well-to-do control methodology with large advantages to hold different problems such as process nonlinearity, constraint handling, stochastic disturbance, tuning etc and is used in many different applications like aerospace, automotive, water, energy etc. In this work, the study of constrained and unconstrained Model Predictive Control (MPC) was described and implemented in multivariable process. The MPC based control system is developed and simulated in MATLAB environment with suitable design parameters. The four-tank non-minimum phase process is taken for investigation. The nonlinear model of the process linearized it for using in the control algorithm and the plant output response is analyzed. From the simulation results, it is clear that, the constrained MPC, exhibits better performance than the unconstrained MPC in terms of tracking, and overshoot, cancellation of oscillation. MPC is a more advanced technique to handle multivariable process with Non-minimum Phase. The work can be extended to give adaptive version for the MPC controller because the adaptive control is one of the most well studied areas in control systems theory. The MPC based control can be designed with constraints for real time implementation.

How to Cite this Article:

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