



He's Multiple-Scale Solution for the Three-Dimensional Nonlinear KH Instability of Rotating Magnetic Fluids

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Received: 03 November 2019 / Accepted: 30 November 2019 / Published: 06 December 2019

ABSTRACT

This paper elucidates a trend in solving nonlinear oscillators of the rotating Kelvin-Helmholtz instability. The system is constituted by the vertical flux or the horizontal flux. He's multiple-scales perturbation methodology has been applied and therefore the system is represented by a generalized homotopy equation. This approach ends up in a periodic answer to a nonlinear oscillator with high nonlinearity. The cubic-quintic nonlinear Duffing equation is obligatory as a condition to uniformly answer. This equation is employed to derive the stability criteria. The transition curves are plotted to investigate the stability image. It's shown that the angular velocity suppresses the instability. The tangential flux plays a helpful role, whereas the vertical field encompasses a destabilizing influence. Within the existence of the rotation, the velocity ratio reduces stability configuration.

Keywords: He's-Multiple-Scale Method, Kelvin-Helmholtz Instability, Nonlinear Stability, Rotating Fluids, Magnetic Fluids.

1 Introduction

During the past years, the study of fluids among the rotation has additional and additional occupied the attention of fluid mechanics Associate degree understanding of rotating fluids is vital once trying to elucidate or predict region or oceanographic phenomena. Evidently, it's the premise to any state of affairs encountered by astrophysicists, and it's vital in respondent technical queries that vary from the centrifuge to the spinning shell. Several mathematical, numerical and empiric research is formed to rotating fluids as an outcome of that the topic presently incorporates a considerable basis, a foundation, what's over the properly trained hydraulics got to perceive despite the very fact that rotating fluids [1].

Effects of rotation on doubly disseminate fluid systems notice application in numerous branches of recent science like biochemistry, earth science, stellar convection, etc. Pearlstein [2] has offered a rapid survey of necessary findings of various researchers in these fields. It's discovered that rotation, in general, enhances the convective

stability of a double-diffusive fluid, except for certain Darcy-Taylor numbers, Darcy-Prandtl numbers and Darcy-Schmidt numbers it performs a destabilizing action for such a fluid.

Another branch of science during which rotation plays a very important role is geology. It's noted that the earth's crust consists of a combination of various kinds of fluids like oil, water, gases, etc. The temperature will increase joined go within. Also, the constant angular velocity of the world with regard to its geographical axis offer increases to force. Therefore, any commit to studying convective currents in energy systems can cause the matter of finding the impact of turnover on the stabilization of a multicomponent fluid, the elements of which might diffuse relative to at least one another [3]. The atmosphere and ocean have such a big amount of fluid-dynamical properties in common that the study of one usually enriches our understanding of the opposite. Geology fluid dynamics is that the subject whose considerations fundamental the basic slashing ideas essential to an associated understanding of



the atmosphere and ocean. In theory, though, earth science, fluid dynamics deals with all conferrment fluid motions [4]. Comprehensive inspection in various situations of the effect of revolution on the stabilization of the atmospheres of stellar and planetary systems has been included and established in the literature. It's necessary to think about that the model fluid is stratified which the angular rate is parallel. In stars and planets, completely different atmosphere section is familiarized in several directions with relevancy the rotation axis whereas gravity continually remains perpendicular to them. In these fluid layer sections; the angular rate vector could also be separated into one part parallel to the fluid layer and another one perpendicular to it. Besides, it's recognized that the field performs a serious role in ionizing atmospheres, and its together determinant in hydromagnetic stability [5]. Rotation and curvature square measure necessary factors that modify stability and flow structure and this have motivated studies in each stability analysis and turbulence modeling. It's usually accepted that the force plays a helpful or destabilizing influence on shift or a turbulent plane flows found in Turbomachinery, geology or astronomy, like shear flows or massive scale vortices [6].

In recent years, the class of disability has been extended by the interest in fluid mechanics flows of electrically conducting fluids among the presence of magnetic fields. This could be the branch of fluid mechanics coupled with the magnetodynamic, and there are problems with hydromagnetic stability as there are problems with mechanical stability. For roughly a century ago, fluid mechanics, stability have been recognized united on the central issues of fluid mechanics. The essential issues of fluid mechanics, stability was recognized and developed within the nineteenth century, notably by John William Strutt, Kelvin, Helmholtz, and Reynolds. To realize a wider understanding of fluid mechanical stability, it's useful to sketch a number of the vital physical mechanisms of instability. Broadly, one could say that instability happens as an outcome of there's some disturbance of the equilibrium of the external

forces, inertia and viscous stress of a fluid. The external forces of interest are buoyancy in a very fluid of variable density, physical phenomenon, magnetohydrodynamic forces, electrohydrodynamic forces, etc. If an important fluid rests higher than lightweight fluid, it's clear that the fluids tend to overturn underneath the action of gravity. An identical instability happens on the free surface of an instrument of liquid once it's rapt down at a standardized acceleration larger than the gravitational acceleration.

The dynamics of wave motion are of nice importance in physical investigations, as wave motion constitutes one among the principal modes of transmission of energy. The energy received from the sun is ancestral by waves during the ether and consequently the energy of sound by airwaves. A wave means that the continual transference of a specific state or from one a part of a medium to a different. This will imply the transference of the medium itself from one area to a different, however, just the propagation through it's of a specific kind, state or condition. The waves because of little periodic motions that surface and akin to the surface of a vast sheet of fluids are referred to as surface waves [7].

The free surface of fluid throughout associate equilibrium state in a very field could also be a plane. If the surface is affected by its equilibrium position at some purpose, the motion can occur within the fluid under the work of some external perturbation. This motion due to be propagated over the whole surface at intervals the sort of waves that are observed as gravity waves. The deformability of the free surface ends up in capillary-gravity waves that rely upon forces tending to come the ill-sharpen surface to its equilibrium plane unless we supply enough energy to overcome the viscosity inertia enabling overshooting of the Laplace overpressure and thus damped oscillations to occur. These cress waves can rely solely slightly on the exchange of substance between the majority and therefore the interface [8].

The KHI has attracted the eye of the many researchers due to its determinant impact on the stabilization of planetary and stellar atmospheres and alternative sensible applications. The study

of the KHI encompasses a protracted history in hydrodynamics. Melcher [9] mentioned the influence of each vertical and horizontal electrical field on Kelvin-Helmholtz for incompressible flow within the presence of physical phenomenon impact. The linear instability of the KH drawback is investigated by Chandrasekhar [10]. He mentioned the impact of natural phenomenon, variable density, rotation and applied the force field on the behavior of the stability. The linear magnetohydrodynamic KHI was studied by Alterman [11] once a system consists of two streaming fluids beneath the gravity and stressed by a horizontal force field. El-Sayed [12, 13] combined the influence of applied electrical fields or oblique electrical fields and uniform rotation on the KHI drawback. He demonstrated that the Coriolis forces, as well as, the streaming velocities, have a destabilizing influence.

The nonlinear development of the KHI in ideal fluids has been dispensed by Drazin [14] and Nayfeh and Saric [15] for the case wherever the amplitude of an unstable wave is uniform in space and growing solely in time. Afterward, Weissman [16] extended the on top of work by treating the case wherever the amplitude of an unstable wave relies on each time and area. Also, He examined the instability of the system close to the crossroads within the parameter area and for regions of linear instability. He found the corrections of Nayfeh and Saric's results [15]. El-Dib [17] studied the nonlinear KHI for magnetic fluids. The fluids are stressed by a relentless tangential field and vertical periodic acceleration. He found that the field, the velocities and also the frequency of the applied periodic force play twin roles within the resonant region.

The nonlinear instability of a surface wave of a streaming magnetic incompressible fluid associate degree a stratified subsonic gas was investigated by Zakaria [18]. KHI in an exceedingly very fluid Layer finite on top of a porous medium and a rigid surface lies below in the existence of field has been investigated by Chavaraddi *et al.* [19]. In [20] Chavaraddi *et al.*, have studied the electrohydrodynamic KHI instability in an exceedingly very fluid layer finite on top of a porous medium and by a rigid

surface below. The target of this paper is to review the impact of Kelvin-Helmholtz separation between two viscous conducting fluids in a very transversal field through a porous medium within the existence of the results of physical phenomenon victimization B-J condition at the interface, among others.

The present work is performed to look at the nonlinear stability behavior of two rotating magnetic fluids that are in relative horizontal motion. The system is subjected by a continuing applied vertically a force field, on the interface through the two fluids. Also, the impact of the horizontal magnetic field is investigated. In the simplest of our information, this downside has not antecedently been investigated. The linearized downside has been incontestable beneath horizontal rotation and magnetic fields by Davalos-Orozco [21]. We've got to target during this work on a weak nonlinear approach that's supported neglecting the nonlinear terms of equations of motion and applying the passable boundary conditions whilst not drop the nonlinear terms. At this step, the dispersion relation should be expanded to combine nonlinear terms. This approach has been successfully applied by El-Dib *et al.* [22]. This approach ends up extending the well-known Chandrasekhar dispersion relation [10] to combine nonlinear terms of the surface elevation. Then the dispersion relation containing nonlinear terms springs in order that it depends not solely on the frequency and therefore the wave number for the wavetrain of the linear solution however also on the amplitude of this wavetrain solution. This conclusion of the nonlinear dispersion relation looks alike those aforesaid in Whitham [23]. Perturbation with He's multiple scale has been applied [24-26] and that we have derived the nonlinear cubic-quintic Duffing equation with real coefficients. Finally, the stability criteria are measure mentioned diagrammatically and analytically.

2 Formulation of the Problem

The interface of flow in equilibrium throughout a field could also be a plane. If the surface is excited from its equilibrium position to some purpose, the motion will come inside the fluid

through the work of some external perturbation. This motion can increase over the complete surface within the variety of waves. The deformability of the free surface ends up in capillary-gravity waves that rely upon forces tending to come the ill-sharpen surface to its equilibrium plane form unless we tend to offer enough energy to beat the body inertia sanctionative overshooting of the mathematician atmospheric pressure and therefore permit damped oscillations to occur. These transversal waves can act between the majority and therefore the interface [10].

The model system adopted consists of two semi-infinite magnetic inviscid incompressible homogenous and identical fluids separated by the horizontal interface $z=0$. The higher and lower densities and magnetic permeableness of the fluids are $\rho^{(1)}, \mu^{(1)}$ and $\rho^{(2)}, \mu^{(2)}$, severally. The fluids are streaming with constant horizontal velocities $u_0^{(1)}$ and $u_0^{(2)}$ wherever the superscripts (1) and (2) check with the higher and therefore the lower fluid, severally. The fluids are impacted by the gravity force associate by uniform rotation concerning the axis with associate angular velocity Ω . The system is subjected by an external horizontal magnetic vertical magnetic field or the vertical magnetic field acting at the negative z-direction. The uniform fields $H^{(1)}$ and $H^{(2)}$ settled at interval 1 and 2, are higher than and blower than the interface, severally.

$t=0$, Owing to the behavior of random surface stress, the interface of separation is slightly malformed from this steady configuration. These random stresses are caused by mechanical or magnetic perturbations. The random stress-induced perturbation within the interface $z=0$ causes a displacement of the fabric particles of the fluid system from their equilibrium locations. This displacement could also be delineated by

$$z = \xi(x, y, t). \quad (1)$$

Let the interface of the fluid is considered as the position of points satisfying the relation $S(x, y, z, t) = z - \xi(x, y, t) = 0$, then the identity

normal vector \underline{n} , to the interface is assumed as

$$\underline{n} = \frac{\nabla S}{|\nabla S|} = \left[-\xi_x \underline{e}_x - \xi_y \underline{e}_y + \underline{e}_z \right] \left[1 + \xi_x^2 + \xi_y^2 \right]^{-1/2}, \quad (2)$$

Where \underline{e}_x , \underline{e}_y and \underline{e}_z refer to the identity vectors in the x-, y- and z-directions.

Maxwell equations for the application of the magnetic fields will be reduced to equations

$$\nabla \cdot (\mu^{(j)} \underline{H}^{(j)}) = 0, \quad (3)$$

$$\nabla \times \underline{H}^{(j)} = \underline{0} \quad j=1,2, \quad (4)$$

where μ referring the magnetic permeability for the fluid phase and the superscript j to the fluid phase. In conformity with the rightness of the quasi-static approximation, a potential function $\phi(x, y, z, t)$ is defined by

$$\underline{H}^{(j)} = -\underline{H}^{(j)} - \nabla \phi^{(j)}. \quad (5)$$

Clearly, the function $\phi(x, y, z, t)$ satisfies Laplace's equation:

$$\nabla^2 \phi^{(j)} = 0. \quad (6)$$

The elemental equations governing the motion, for a bulk of magnetic fluid phases, are written within the type of rotation $\underline{\Omega}$ in the form

$$\rho \left[\frac{\partial \underline{V}}{\partial t} + (\underline{V} \cdot \nabla) \underline{V} + 2(\underline{\Omega} \times \underline{V}) - \frac{1}{2} \rho \nabla (|\underline{\Omega} \times \underline{r}|^2) \right] = -\nabla p - \rho g \underline{e}_z. \quad (7)$$

Associated with the continuity equation:

$$\nabla \cdot \underline{V} = 0, \quad (8)$$

where p is the hydrodynamic pressure, ρ is the density of the flow and $\underline{V} = (u, v, w)$ represents the velocity of the fluid. The term $2(\underline{\Omega} \times \underline{V})$ in equation (7) represents the Coriolis force and the term $\frac{1}{2} \rho \nabla (|\underline{\Omega} \times \underline{r}|^2)$ indicates to the centrifugal force where \underline{r} refers to the position vector of any point of the fluid. The Coriolis term is beneficially introduced in the equation of motion in the state of the rotating frame of reference when the angular velocity Ω is uniform.

The total pressure is defined as

$$\pi = p - \frac{1}{2} \rho \Omega^2 r^2 - \frac{1}{2} \mu H^2. \quad (9)$$

The pure equilibrium configuration gives,

$$\begin{aligned} \pi_0^{(j)}(x, y, z, t) \\ = -\rho^{(j)} g z - 2\rho^{(j)} \Omega u_0^{(j)} y + \frac{1}{2} \rho^{(j)} \Omega^{(j)2} r^2 + \lambda_0^{(j)}; \\ j = 1, 2, \end{aligned} \quad (10)$$

where $\lambda_0^{(j)}$ are the constants of integration. The balance of the normal stress tensor at the interface leads to:

$$\begin{aligned} \lambda_0^{(2)} - \lambda_0^{(1)} \\ = 2\Omega(\rho^{(2)} u_0^{(2)} - \rho^{(1)} u_0^{(1)}) y - \frac{1}{2} \Omega^2 (\rho^{(2)} - \rho^{(1)}) r^2 \\ + \frac{1}{2} (\mu^{(2)} H^{(2)2} - \mu^{(1)} H^{(1)2}). \end{aligned} \quad (11)$$

2.1 Boundary Conditions

It is convenient to enclose that $\phi(x, y, z, t)$ is a finite function presented due to the perturbed interface and far from the interface, its influence vanishes. Therefore, both the partial derivative for $\phi(x, y, z, t)$ with respect to x, y and z must vanish as $z \rightarrow \pm\infty$. At the dividing surface, the following boundary conditions must be satisfied:

(i) The continuity of the normal component of the magnetic displacement at the surface of separation is

$$\underline{n} \cdot (\mu^{(1)} \underline{H}^{(1)} - \mu^{(2)} \underline{H}^{(2)}) = 0, \quad z = \xi. \quad (12)$$

This leads to

$$\begin{aligned} \xi_x (\mu^{(1)} \varphi_x^{(1)} - \mu^{(2)} \varphi_x^{(2)}) + \xi_y (\mu^{(1)} \varphi_y^{(1)} - \mu^{(2)} \varphi_y^{(2)}) \\ - (\mu^{(1)} \varphi_z^{(1)} - \mu^{(2)} \varphi_z^{(2)}) = 0, \quad z = \xi. \end{aligned} \quad (13)$$

(ii) The continuity of the tangential component of the magnetic field is assumed at the interface $z = \xi$. Thus,

$$\underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)}) = 0, \quad z = \xi. \quad (14)$$

It yields that

$$\begin{aligned} (\xi_y - \xi_x) \left[(\underline{H}^{(1)} - \underline{H}^{(2)}) + (\varphi_z^{(1)} - \varphi_z^{(2)}) \right] \\ + (\varphi_y^{(1)} - \varphi_y^{(2)}) - (\varphi_x^{(1)} - \varphi_x^{(2)}) = 0, \quad z = \xi. \end{aligned} \quad (15)$$

(iii) At the dividing surface, the velocity field u, v and w are subject to the following boundary

condition:

$$\begin{aligned} w^{(j)} = \xi_t + \xi_x (u_0^{(j)} + u^{(j)}) + \xi_y v^{(j)}, \\ j = 1, 2, \quad z = \xi. \end{aligned} \quad (16)$$

This equation comes across the given tool character of the dividing surface. Far from the interface, the fluid velocity vanishes. Thus,

$$\underline{V}^{(j)}(x, y, \pm\infty, t) = 0. \quad (17)$$

(iv) At the interface, the hydro stress and the magnetic forces must be balanced. The composition of these depends on the hydrodynamic pressure, surface tension effects and magnetic stresses. The magnetic stresses result from the magnetization forces [27] and [28]. Thus, the normal composition of the stress tensor σ_{ij} is discontinuous at the interface by the surface tension, i.e.

$$\underline{n} \cdot (\underline{F}^{(1)} - \underline{F}^{(2)}) = \sigma_T \underline{n}, \quad z = \xi, \quad (18)$$

where \underline{F} is the force vector acting on the interface, the surface tension coefficient is denoted by the parameter σ_T and σ_{ij} given by

$$\sigma_{ij} = -\pi \delta_{ij} + \mu H_i H_j - \frac{1}{2} \mu H^2 \delta_{ij}, \quad (19)$$

where π is the total pressure.

3 Outlined Steps of Solutions

The linear stability analysis as given by Chandrasekhar [10] depends on quit the nonlinear terms from the equations of motion furthermore as from the boundary conditions. Therefore, a dispersion relation ought to have got without nonlinear terms. The concept of the weak nonlinear approach is slightly departed from the dimensionality technique. At this state, the nonlinear downside can contain the linear description with some further terms that build a correction of the most solution. The nonlinear description given here depends on quit the nonlinear terms from the equations of motion and applying the agreeable boundary conditions involving the nonlinearity. Therefore, the Chandrasekhar dispersion relation ought to be elongated to combine the nonlinear terms.

To solve the linear form of the equations of motion for the fluid phases, three-dimensional finite disturbances are introduced to the

boundary worth downside. We tend to suppose there's a standardized monochromatic wave train propagating on the interface such that:

$$\xi(x, y, t) = \gamma e^{ik\xi - i\omega t} + c.c., \quad (20)$$

where c.c. represents complex conjugate for preceding terms, the amplitude γ is an integration constant which locates the behavior of the perturbed of the interface, ω is the frequency of the perturbed and $i = \sqrt{-1}$. The spatial variable is $\xi = (k_x x + k_y y) / k$. The

wavenumber k is defined as $k = \sqrt{k_x^2 + k_y^2}$, which is assumed to be real and positive. Similar Fourier criteria, we may suppose that the bulk solutions are of the form:

$$V(x, y, z, t) = \hat{V}(z) e^{ik\xi - i\omega t}, \quad (21)$$

$$\pi(x, y, z, t) = \hat{\pi}(z) e^{ik\xi - i\omega t}, \quad (22)$$

$$\phi(x, y, z, t) = \hat{\phi}(z) e^{ik\xi - i\omega t}. \quad (23)$$

In view of equations (21) through (23), the components of equations (7) and (8) take the form:

$$i\rho(k_x u_0 - \omega)\hat{u} - 2\rho\Omega\hat{v} = -ik_x \hat{\pi}, \quad (24)$$

$$i\rho(k_x u_0 - \omega)\hat{v} + 2\rho\Omega\hat{u} = -ik_y \hat{\pi}, \quad (25)$$

$$i\rho(k_x u_0 - \omega)\hat{w} = -\frac{d\hat{\pi}}{dz}, \quad (26)$$

$$ik_x \hat{u} + ik_y \hat{v} = -\frac{d\hat{w}}{dz}. \quad (27)$$

Equations (24), (25) and (27) yield:

$$\hat{\pi} = \frac{-i\rho(k_x u_0 - \omega)}{k^2} \left(1 - \frac{4\Omega^2}{(k_x u_0 - \omega)^2} \right) \frac{d\hat{w}}{dz}. \quad (28)$$

Combining equations (26) and (28), we obtain:

$$\frac{d^2 \hat{w}}{dz^2} - q^2 \hat{w} = 0, \quad (29)$$

where

$$q^{(j)} = k \left(1 - \frac{4\Omega^2}{(k_x u_0^{(j)} - \omega)^2} \right)^{-1/2} \quad j = 1, 2. \quad (30)$$

Using the kinematic boundary condition (16), the continuity equation (8) and applying the Maclaurin series at $\xi = 0$, we obtain the solution of equation (29) gives the perturbation of the vertical velocity component in the form

$$w^{(1)}(x, y, z, t) = \frac{i(k_x u_0^{(1)} - \omega)\xi}{(1 - q^{(1)}\xi)} e^{-q^{(1)}z}; \quad z > 0, \quad (31)$$

$$w^{(2)}(x, y, z, t) = \frac{i(k_x u_0^{(2)} - \omega)\xi}{(1 + q^{(2)}\xi)} e^{q^{(2)}z}; \quad z < 0. \quad (32)$$

In light of the equation (28), the pressure distributions in the two fluid phases are:

$$\pi^{(1)}(x, y, z, t) = -\frac{\rho^{(1)}(k_x u_0^{(1)} - \omega)^2 \xi}{q^{(1)}(1 - q^{(1)}\xi)} e^{-q^{(1)}z}; \quad z > 0, \quad (33)$$

$$\pi^{(2)}(x, y, z, t) = \frac{\rho^{(2)}(k_x u_0^{(2)} - \omega)^2 \xi}{q^{(2)}(1 + q^{(2)}\xi)} e^{q^{(2)}z}; \quad z < 0. \quad (34)$$

To derive the solution for the magnetic function $\phi(x, y, z, t)$, we insert (23) into the Laplace equation (6), for using both conditions (13) and (15), the resulting solution is

$$\phi^{(1)}(x, y, z, t) = H^{(1)} \left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}} \right) \frac{\xi}{(1 - k\xi)} e^{-kz}, \quad z > 0, \quad (35)$$

$$\phi^{(2)}(x, y, z, t) = -H^{(2)} \left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}} \right) \frac{\xi}{(1 + k\xi)} e^{kz}, \quad z < 0, \quad (36)$$

The distributions for the fluid velocity components $u^{(j)}$ and $v^{(j)}$, the hydrodynamic pressure $\pi^{(j)}$ and the magnetic functions $\phi^{(j)}$, both contain nonlinear terms as a function in the elevation amplitude ξ . This nonlinear depiction is due to the application of nonlinear boundary conditions. As the nonlinear terms are dropped, the linear profile got, and it is equivalent to those obtained earlier by Chandrasekhar [10] and by El-Sayed [13] for a general oblique electric field.

A solution to the equation of motion cited before is accomplished by utilizing the convenient nonlinear precondition. At the interface between fluids, the fluids and therefore the magnetic flux should be balanced. The elements of those stresses comprise fluid mechanics, pressure, velocities, stresses, physical phenomenon stresses, rotation velocities, and magnetic stresses.

4 Nonlinear dispersion relation

In order to derive the nonlinear characteristic equation, we have a tendency to shall use an equivalent procedure utilized by Chandrasekhar [10]. To get the linear dispersion relation, however, while not ignoring the nonlinear terms

of the elevation parameter ξ , we have a tendency to substitute from equations (31) through (36) into equation (18). Hence, we have:

$$\begin{aligned} & \frac{\rho^{(1)}}{q^{(1)}} (k_x u_0^{(1)} - \omega)^2 (1 - \xi q^{(1)})^{-1} \xi \\ & + \frac{\rho^{(2)}}{q^{(2)}} (1 + \xi q^{(2)})^{-1} (k_x u_0^{(2)} - \omega)^2 \xi \\ & + (\rho^{(1)} - \rho^{(2)}) g \xi - k^2 \sigma_T \xi (1 - k^2 \xi^2)^{-3/2} \\ & + k \sigma_{H\perp} \xi (1 - k^2 \xi^2)^{-1} \left[1 + Jk \left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}} \right) \xi \right] = 0. \end{aligned} \quad (37)$$

where the notion $\sigma_{H\perp}$ is named the performance of the conventional force field that given by

$$\sigma_{H\perp} = H^{(1)} H^{(2)} \frac{(\mu^{(1)} - \mu^{(2)})^2}{\mu^{(1)} + \mu^{(2)}}. \quad (38)$$

On the other side, if the system is impacted by a horizontal magnetic field such that $H = H_0 e_z$

then the same characteristic equation will be obtained except that the term $\sigma_{H\perp}$ should be replaced by

$$\sigma_{H\parallel} = -H_0^2 \frac{(\mu^{(1)} - \mu^{(2)})^2}{\mu^{(1)} + \mu^{(2)}}, \quad (39)$$

where the notion $H\parallel$ denotes the horizontal magnetic term. The parameter $J = -1$ in the stage of the vertical field, while $J = 1$ referring to the application of a tangential magnetic field.

Equation (37) represents an amplification of the famous Chandrasekhar's dispersion relation [10], by inclusive some higher-order terms of the amplitude parameter ξ . It is more general than those obtained by Davalos-Orozco [21] and El-Sayed [12, 13]. In the case of non-rotated fluids, it reduces for streaming electrified viscous fluids over porous media by Mohamed [29].

The linear form of the system (37) is performed as all the second powers or higher are tends to zero

$$\left[\begin{array}{l} \frac{\rho^{(1)}}{q^{(1)}} (k_x u_0^{(1)} - \omega)^2 + \frac{\rho^{(2)}}{q^{(2)}} (k_x u_0^{(2)} - \omega)^2 \\ + (\rho^{(1)} - \rho^{(2)}) g + k \sigma_{H\perp} - k^2 \sigma_T \end{array} \right] \gamma = 0, \quad (40)$$

which is in the transcendental form. In the light of a slow rotating fluid, the square root (30) can be expanded applying the binomial theory to become

$$q^{(j)} \cong k \left(1 + \frac{2\Omega^2}{(k_x u_0^{(j)} - \omega)^2} \right). \quad (41)$$

At this state, the primary dispersion relation is found to be

$$D(\omega, k_x, k_y) \equiv (\rho^{(1)} + \rho^{(2)}) ((\omega - k_x u^*)^2 - 2\Omega^2 + \sigma_\omega^2), \quad (42)$$

where the following notion is used:

$$u^* = \frac{\rho^{(1)} u_0^{(1)} + \rho^{(2)} u_0^{(2)}}{\rho^{(1)} + \rho^{(2)}}. \quad (43)$$

The parameter σ_ω^2 is the surface frequency for KHI in the case of no rotation, such that:

$$\sigma_\omega^2 = \frac{1}{\rho^{(1)} + \rho^{(2)}} \left[\begin{array}{l} k(\rho^{(1)} - \rho^{(2)})g + k^2 \sigma_{H\perp} \\ -k^3 \sigma_T + \frac{k_x^2 \rho^{(1)} \rho^{(2)}}{\rho^{(1)} + \rho^{(2)}} (u_0^{(1)} - u_0^{(2)})^2 \end{array} \right]. \quad (44)$$

We first study equation (42) and so come back to equation (37) to include higher order dispersive effects. It ought to be noticed that equation (42) represents the linear dispersion relation that's glad by values of ω , k_x and k_y . Within the equation (42), ω seems like a square term solely, whereas, within the right-hand facet, it is real. Thus, the values of being either real or imagined. Once is imagined, an instability is expressed through the dependence of ω^2 on the wavenumber k . However, stability happens once the angular rate satisfies the subsequent relation:

$$\Omega^2 > \frac{1}{2} \sigma_\omega^2. \quad (45)$$

In the case of no rotation, i.e. in the limiting case as $\Omega \rightarrow 0$ the stability is found when the magnetic field satisfies the following relation:

$$\sigma_{H\perp} < \frac{1}{k^2} \left[\begin{array}{l} k^3 \sigma_T - k(\rho^{(1)} - \rho^{(2)})g \\ - \frac{k_x^2 \rho^{(1)} \rho^{(2)}}{\rho^{(1)} + \rho^{(2)}} (u_0^{(1)} - u_0^{(2)})^2 \end{array} \right]. \quad (46)$$

The on top of dispersion relation can scale back to those obtained by Davalos-Orozco [21] for streaming inviscid fluids underneath horizontal rotation and magnetic fields. Within the case of no field of force, the dispersion relation reduces

to the well-established strictly the inviscid dispersion relation given by Chandrasekhar [10] and reduces by El-Sayed [13] for streaming electrified inviscid rotation fluids.

5 Construction of the homotopy nonlinear dispersion relation

In this portion, we have a tendency to affect the examination of the influence of finite angular speed on the steadiness behavior. During this case, we have a tendency to could use the growth procedure obtained formally by the homotopy perturbation [30-35]. By introducing the homotopy parameter $\delta \in [0,1]$, the homotopy equation admire equation (37) is created within the type

$$\begin{aligned} & \frac{\rho^{(1)}}{q^{(1)}} (k_x u_0^{(1)} - \omega)^2 (1 - \delta \xi q^{(1)})^{-1} \xi \\ & + \frac{\rho^{(2)}}{q^{(2)}} (1 + \delta \xi q^{(2)})^{-1} (k_x u_0^{(2)} - \omega)^2 \xi \\ & + (\rho^{(1)} - \rho^{(2)}) g \xi - k^2 \sigma_T \xi (1 - \delta k^2 \xi^2)^{-3/2} \\ & + k \sigma_{H\perp} \xi (1 - \delta k^2 \xi^2)^{-1} \left[1 + \delta J k \left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}} \right) \xi \right] = 0. \end{aligned} \quad (47)$$

The homotopy construction built for the nonlinear system, composed of the primary part arises when $\delta \rightarrow 0$, which can be easily solved, the reaming parts including the nonlinear terms. It is noted that the above homotopy equation (47) will become the original equation (37) as $\delta \rightarrow 1$. By sought $\delta \rightarrow 1$ in the final approximate solution for the perturbation equations of equation (47) will become the solution of the original one.

In view of the definitions (41) and (42), the application of the binomial theorem leads to expanding, the nonlinear dispersion relation (47) as an infinite power series in δ . Collecting the identical powers in δ , the above nonlinear dispersion relation may be rearranged as

$$\begin{aligned} & L(\partial_t, \partial_\zeta) \xi + \delta (\omega_{11} \xi^2 + \omega_{12} \xi^3) \\ & + \delta^2 (\omega_{21} \xi^3 + \omega_{22} \xi^4 + \omega_{23} \xi^5) + O(\delta^3) = 0, \end{aligned} \quad (48)$$

where L is a linear operator involving the partial derivative of ∂_t and ∂_ζ , $D(\omega, k) \xi$ represents

the primary dispersion relation which is corresponding to the differential equation:

$$L(\partial_t, \partial_\zeta) \xi(\zeta, t) = 0, \quad (49)$$

The constant coefficients ω_{ij} that show in equation (48) are defined as:

$$\begin{aligned} \omega_{11} &= k \rho^{(1)} (k_x u_0^{(1)} - \omega)^2 - k \rho^{(2)} (k_x u_0^{(2)} - \omega)^2 \\ &+ J k^3 \sigma_{H\perp} \frac{(\mu^{(1)} - \mu^{(2)})}{(\mu^{(1)} + \mu^{(2)})}, \\ \omega_{12} &= k^4 \left(\sigma_{H\perp} - \frac{3}{2} k \sigma_T \right), \\ \omega_{21} &= k^2 \rho^{(1)} (k_x u_0^{(1)} - \omega)^2 + k^2 \rho^{(2)} (k_x u_0^{(2)} - \omega)^2 \\ &+ 2k^2 \Omega^2 (\rho^{(1)} + \rho^{(2)}), \\ \omega_{22} &= J k^5 \sigma_{H\perp} \left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}} \right), \\ \omega_{23} &= k^6 \left(\sigma_{H\perp} - \frac{15}{8} k \sigma_T \right). \end{aligned} \quad (50)$$

5.1 The modulation for the linear dispersion relation of the He's-multiple scale technology

In the linear approximation, the amplitude γ is defined to be constant. Once nonlinearity is incorporated, it's treated as slowly varied operate of area and time. The existence of the harmonic wavetrain during a dispersive medium and also the correspondence between the wave number and frequency ends up in the dispersion relation (40).

Introducing the wave parameter $\theta = k\zeta - \omega t$ such that $\xi(\zeta, t)$ becomes $\xi(\theta)$ and

$$\begin{aligned} \frac{\partial \xi}{\partial t} &= \frac{\partial \theta}{\partial t} \frac{d\xi}{d\theta} = -\omega \frac{d\xi}{d\theta}, \\ \frac{\partial \xi}{\partial \zeta} &= \frac{\partial \theta}{\partial \zeta} \frac{d\xi}{d\theta} = k \frac{d\xi}{d\theta}. \end{aligned} \quad (51)$$

At this stage, we may introduce a modulation to the problem so that the linear dispersion relation $D(\omega, k_x, k_y)$ indicates the slowly modulated wavetrain [36]. To do this, we may use the expansion procedure obtained formerly by the procedure of multiple scaling [37]. The underlying idea of the procedure of multiple scales is to make the expansion perform the solution of the problem, not only a function of one independent variable but also as a function

of two or additional independent variables which are referred to as scales. The independent variable θ can be alternative, independent variables, that is;

$$\theta_n = \delta^n \theta; \quad n = 0, 1, 2. \quad (52)$$

Thus, defining θ_0 as the variables appropriate to the fast variations and θ_1, θ_2 are the slow variables. The differential operators can now be expressed as the derivative expansions:

$$\frac{d}{d\theta} = \frac{\partial}{\partial \theta_0} + \delta \frac{\partial}{\partial \theta_1} + \delta^2 \frac{\partial}{\partial \theta_2} + \dots \quad (53)$$

The analysis then follows the usual perturbation procedure. In view of the above alternative, independent variables, the operator L have extended to:

$$L \left[-i\omega, ik + i\delta \frac{\partial}{\partial \theta_1} + i\delta^2 \frac{\partial}{\partial \theta_2} + \dots \right] \xi = 0. \quad (54)$$

Using Taylor's theorem about $D(\omega, k_x, k_y)$ and retains only terms up to $O(\delta^2)$. Thus, we have

$$L\xi = D\xi + i\delta D' \frac{\partial}{\partial \theta_1} \xi + \delta^2 \left(iD' \frac{\partial}{\partial \theta_2} - D'' \frac{\partial^2}{\partial \theta_1^2} \right) \xi, \quad (55)$$

where the notations D' and D'' are

$$D' = \left(\frac{\partial}{\partial \omega} - \frac{1}{k} (k_x + k_y) \frac{\partial}{\partial k} \right) D(\omega, k), \text{ and} \quad (56)$$

$$D'' = \left(\frac{\partial}{\partial \omega} - \frac{1}{k} (k_x + k_y) \frac{\partial}{\partial k} \right)^2 D(\omega, k).$$

Expressing the expansion of the expanded operator (55) into equation (48) we get,

$$L_0 \xi + \delta \left(iD' \frac{\partial \xi}{\partial \theta_1} + \omega_{11} \xi^2 + \omega_{12} \xi^3 \right) + \delta^2 \left(iD' \frac{\partial \xi}{\partial \theta_2} - D'' \frac{\partial^2 \xi}{\partial \theta_1^2} + \omega_{21} \xi^3 + \omega_{22} \xi^4 + \omega_{23} \xi^5 \right) + \dots = 0. \quad (57)$$

Now, using the homotopy expansion for the variable ξ so that it may be developed in the form

$$\xi(\theta; \delta) = \sum_{n=0}^2 \delta^n \xi_n(\theta_0, \theta_1, \theta_2) + O(\delta^3). \quad (58)$$

Substituting equation (58) into equation (55), then equating similar powers of δ on both sides we obtain the following perturbed equations:

$$\delta^0 : L_0 \xi_0 = 0, \quad (59)$$

$$\delta^1 : L_0 \xi_1 = -iD' \frac{\partial}{\partial \theta_1} \xi_0 - \omega_{11} \xi_0^2 - \omega_{12} \xi_0^3, \quad (60)$$

$$\delta^2 : L_0 \xi_2 = -iD' \frac{\partial}{\partial \theta_1} \xi_1 - \left(iD' \frac{\partial}{\partial \theta_2} - D'' \frac{\partial^2}{\partial \theta_1^2} \right) \xi_0 - 2\omega_{11} \xi_0 \xi_1 - 3\omega_{12} \xi_0^2 \xi_1 - \omega_{21} \xi_0^3 - \omega_{22} \xi_0^4 - \omega_{23} \xi_0^5. \quad (61)$$

where the operator L_0 is defined as

$$L_0 \equiv (-\omega, k) \frac{\partial}{\partial \theta_0}. \quad (62)$$

In the smallest order approximation and in the view of (20), we may write the solution of equation (59) as:

$$\xi_0(\theta_0, \theta_1, \theta_2) = \gamma(\theta_1, \theta_2) e^{i\theta_0} + \bar{\gamma}(\theta_1, \theta_2) e^{-i\theta_0}. \quad (63)$$

We may now solve equations (60) and (61) in turn, noting that for each order greater than the zero, the non-secular conditions may be gotten by setting coefficient of $e^{\pm i\theta_0}$ equal to zero in equations (59), (60) and (61). Thus, we have besides equation (59) the following conditions:

$$iD' \frac{\partial \gamma}{\partial \theta_1} = -3\omega_{12} \gamma^2 \bar{\gamma}. \quad (64)$$

Without secular terms, the solution of equation (63) is uniformly valid and may be written in the form

$$\xi_1(\theta_0, \theta_1, \theta_2) = -\frac{\omega_{11}}{D_2} \gamma^2(\theta_1, \theta_2) e^{2i\theta_0} - \frac{\omega_{12}}{D_3} \gamma^3(\theta_1, \theta_2) e^{3i\theta_0} + c.c., \quad (65)$$

where the non-zero denominator $D_n(\omega, k)$

; $n = 0, 1, 2, \dots$ is defined as

$$D_n \equiv D(n\omega, nk). \quad (66)$$

The vanishing of the divisor refers to the second-harmonic resonance. In general, harmonic resonance could exist if (ω, k) and (n, nk) satisfies constant dispersion relation [37]. Once resonance happens, we discover that each the surface distortion and also the exited volume pulsation bear modulation. At actual resonance, solely modulation occurs and also the modulations are monotonic functions of time; the amount pulsation will increase because it attracts energy from the surface distortion mode. Near, but not at, resonance, energy is changed cyclically between the surface and volumetrically

modes; the oscillations during this case expertise each amplitude and phase modulation. However, the second-harmonic resonance happens at the points derived by solving the equations $D_2 = 0$ and $D_3 = 0$. They're going to satisfy the subsequent equations, respectively:

$$2k^3\sigma_T + kg(\rho^{(1)} - \rho^{(2)}) - 3\Omega^2(\rho^{(1)} + \rho^{(2)}) = 0, \quad (67)$$

$$9k^3\sigma_T + 3kg(\rho^{(1)} - \rho^{(2)}) - 8\Omega^2(\rho^{(1)} + \rho^{(2)}) = 0. \quad (68)$$

Employing (63) and (65) into (61) and removing the secular terms which leads to catching the following condition:

$$\left(iD' \frac{\partial}{\partial \theta_2} - D'' \frac{\partial^2}{\partial \theta_1^2} \right) \gamma = \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} \right) \gamma^2 \bar{\gamma} + \left(\frac{3\omega_{12}^2}{D_3} - 10\omega_{23} \right) \gamma^3 \bar{\gamma}^2. \quad (69)$$

With this condition and in view of (65), the second-order solution has the form

$$\begin{aligned} \xi_2 = & \frac{6}{D_0} \left(\frac{\omega_{11}\omega_{12}}{D_2} - \omega_{22} \right) \gamma^2 \bar{\gamma}^2 \\ & + \frac{2}{D_2} \left(\frac{\omega_{11}\omega_{12}}{D_3} - 2\omega_{22} \right) \gamma^3 \bar{\gamma} e^{2i\theta_0} \\ & + \frac{1}{D_3} \left(\frac{2\omega_{11}^2\gamma^3}{D_2} - \frac{3\omega_{12}^2}{D_3} \gamma^4 \bar{\gamma} - \omega_{21}\gamma^3 - \omega_{23}\gamma^4 \bar{\gamma} \right) e^{3i\theta_0} \\ & + \frac{1}{D_4} \left(\frac{2\omega_{11}\omega_{12}}{D_3} + \frac{3\omega_{11}\omega_{12}}{D_2} - \omega_{22} \right) \gamma^4 e^{4i\theta_0} \\ & + \frac{1}{D_5} \left(\frac{3\omega_{12}^2}{D_3} - \omega_{23} \right) \gamma^5 e^{5i\theta_0} + c.c., \quad (70) \end{aligned}$$

The second-order approximate solution may be performed by employing (63), (65) and (70) into (58) and let $\delta \rightarrow 1$, we obtain

$$\begin{aligned} \xi(\theta) = & \gamma e^{i\theta} + \frac{6}{D_0} \left(\frac{\omega_{11}\omega_{12}}{D_2} - \omega_{22} \right) \gamma^2 \bar{\gamma}^2 \\ & + \frac{1}{D_2} \left[\left(\frac{2\omega_{11}\omega_{12}}{D_3} - 4\omega_{22} \right) \gamma^3 \bar{\gamma} - \omega_{11}\gamma^2 \right] e^{2i\theta} \\ & + \frac{1}{D_3} \left[\left(\frac{2\omega_{11}^2}{D_2} - \omega_{21} - \omega_{12} \right) \gamma^3 - \left(\frac{3\omega_{12}^2}{D_3} + \omega_{23} \right) \gamma^4 \bar{\gamma} \right] e^{3i\theta} \\ & + \frac{1}{D_4} \left(\frac{2\omega_{11}\omega_{12}}{D_3} + \frac{3\omega_{11}\omega_{12}}{D_2} - \omega_{22} \right) \gamma^4 e^{4i\theta} \\ & + \frac{1}{D_5} \left(\frac{3\omega_{12}^2}{D_3} - \omega_{23} \right) \gamma^5 e^{5i\theta} + c.c., \quad (71) \end{aligned}$$

In the above approximate solution, the amplitude $\gamma(\theta)$ is still unknown and needs to be evaluated for the complete solution.

5.2 Derivation of the complete wave-solution

To formulate the second-order complete solution, the amplitude equation will be constructed and solved. This requires to combine both the two solvability conditions (64) and (69).

If we remove the term containing $\frac{\partial^2}{\partial \theta_1^2}$ from the

condition (69), with the help of the condition (64), then the second-order solvability condition (69) will reduce to the form:

$$\begin{aligned} iD' \frac{\partial \gamma}{\partial \theta_2} = & \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} \right) \gamma^2 \bar{\gamma} \\ & + \left(3\omega_{12}^2 \left(\frac{1}{D_3} - \frac{3D''}{D'^2} \right) - 10\omega_{23} \right) \gamma^3 \bar{\gamma}^2. \quad (72) \end{aligned}$$

Combine the first-order solvability condition (64) with the condition (72) and return to the original variable θ . This can be accomplished by multiplying condition (64) by δ and adding to condition (69) multiply, by δ^2 , then letting $\delta \rightarrow 1$, the following cubic–quintic nonlinear Landau equation is obtained:

$$\begin{aligned} iD' \frac{d\gamma}{d\theta} = & \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} - 3\omega_{12} \right) \gamma^2 \bar{\gamma} \\ & + \left(3\omega_{12}^2 \left(\frac{1}{D_3} - \frac{3D''}{D'^2} \right) - 10\omega_{23} \right) \gamma^3 \bar{\gamma}^2. \quad (73) \end{aligned}$$

The polar form solution may be applied to solve the above nonlinear equation by introducing the following form:

$$\gamma(\theta) = \alpha(\theta) e^{i\beta(\theta)}, \quad (74)$$

with real $\alpha(\theta)$ and $\beta(\theta)$. Employing (74) into (73) and separating the real and imaginary parts, the solution of the resulting equations leads to

$$\gamma(\theta) = \alpha_0 e^{-i\varpi\theta}, \quad (75)$$

where α_0 and β_0 are arbitrary constants, while the frequency ϖ is given by

$$\varpi = \frac{1}{D'} \left[\begin{array}{l} \beta_0 + \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} - 3\omega_{12} \right) \alpha_0^2 \\ + \left(3\omega_{12}^2 \left(\frac{1}{D_3} - \frac{3D''}{D'^2} \right) - 10\omega_{23} \right) \alpha_0^4 \end{array} \right]. \quad (76)$$

Inserting (75) into the wave-solution (71) we get

$$\begin{aligned} \xi(\theta) &= \frac{6}{D_0} \left(\frac{\omega_{11}\omega_{12}}{D_2} - \omega_{22} \right) + 2\alpha_0 \cos(1-\varpi)\theta \\ &+ \frac{2}{D_2} \left[\left(\frac{2\omega_{11}\omega_{12}}{D_3} - 4\omega_{22} \right) \alpha_0^2 - \omega_{11} \right] \alpha_0^2 \cos 2(1-\varpi)\theta \\ &+ \frac{2}{D_3} \left[\left(\frac{2\omega_{11}^2}{D_2} - \omega_{21} - \omega_{12} \right) - \left(\frac{3\omega_{12}^2}{D_3} + \omega_{23} \right) \alpha_0^2 \right] \alpha_0^3 \\ &\times \cos 3(1-\varpi)\theta + \frac{2}{D_4} \left(\frac{2\omega_{11}\omega_{12}}{D_3} + \frac{3\omega_{11}\omega_{12}}{D_2} - \omega_{22} \right) \alpha_0^4 \\ &\times \cos 4(1-\varpi)\theta + \frac{2}{D_5} \left(\frac{3\omega_{12}^2}{D_3} - \omega_{23} \right) \alpha_0^5 \cos 5(1-\varpi)\theta. \end{aligned} \quad (77)$$

This is the complete solution describing the surface deflection of the KHI under the action of the small axial rotation. In the following sub-section, the numerical picture will be illustrated.

5.3 Numerical illustrations for the complete wave-solution

In the present sub-section, we graphically the surface deflection $\xi(\zeta, t)$ as obtained in (77). Before we proceed to the numerical illustration for the function $\xi(\zeta, t)$ of the surface wave, it is useful to introduce the following dimensionless forms:

The characteristic length $L = (u_0^{(2)2} / g)$, the characteristic time $t = (L / u_0^{(2)})$, and other dimensionless quantities are given by

$$k = k^* L^{-1}, \quad \omega = \omega^* t^{-1}, \quad \sigma_H = \sigma_H^* (\rho^{(2)} u_0^{(2)2}),$$

$$k_x = k^* L^{-1}, \quad \Omega = \Omega^* t^{-1}, \quad \sigma_T = \sigma_T^* (L \rho^{(2)} u_0^{(2)2}).$$

where $\rho = \rho^{(1)} / \rho^{(2)}$, $V = u_0^{(1)} / u_0^{(2)}$ and the superposed asterisk refers to the dimensionless quantity, which will be omitted later for simplicity.

The calculations are done for a system having

$$\rho = 1.5, \quad k_x = 0.5, \quad k = 0.3, \quad \sigma_T = 1.2, \quad \omega = 1, \quad \mu = 0.46, \quad \sigma_{H\perp} = 1.5,$$

$$\Omega = 0.1 \text{ and } \alpha_0 = 0.1, \beta_0 = 0.5.$$

The result in of the numerical calculations is displayed in the 3D graphs in Fig. (1)–Fig. (6). It is ascertained that the rise within the stratified V will increase the amplitude of the surface deflection $\xi(\zeta, t)$ that indicates the destabilizing action in the rise of V . The impact of the angular rate Ω on the surface waves has been displayed within the graph of Fig. (5). It is shown that the rise of Ω results in decreasing within the amplitude of the surface deflection. On the opposite hand, the rise within the vertical field causes increased within the amplitude of the surface wave.

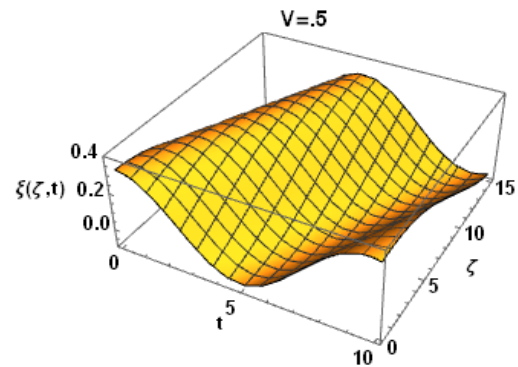


Figure 1: The elevation of the surface wave in the case of the $V = 0.5$

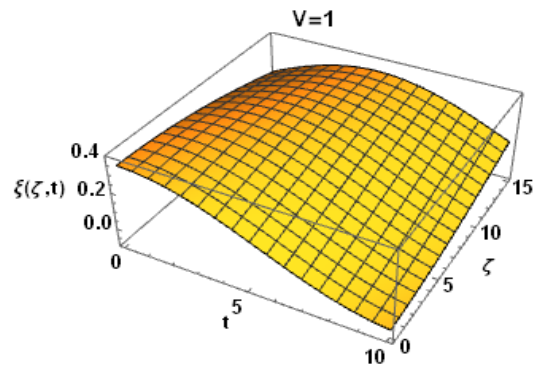


Figure 2: Similar graph as given in Fig. (1) except that $V = 1$. stratified velocity.

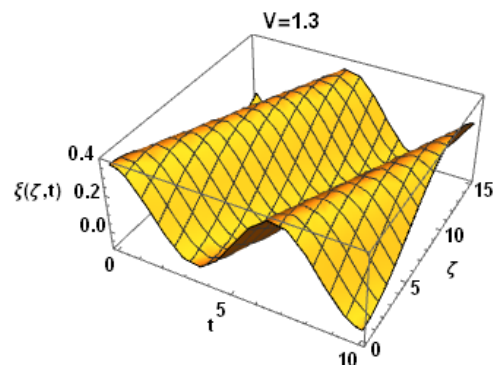


Figure 3: Similar graph as given in Fig. (1) except that $V = 1.3$

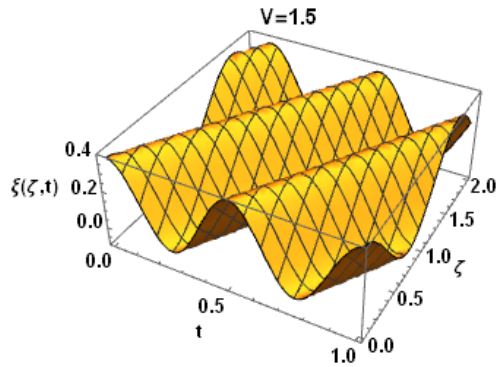


Figure 4: Similar graph as given in Fig.(1) except that $V = 1.5$

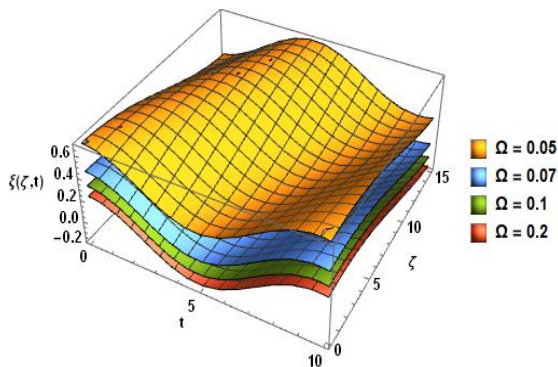


Figure 5: The distribution of the angular frequency Ω on the surface wave similar system as given in Fig. (1)

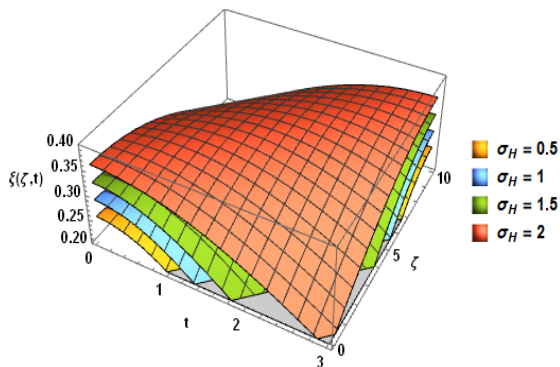


Figure 6: The distribution of the normal magnetic field on the surface wave for similar system as in Fig. (1).

5.4 Derivation of the Duffing equation and the stability criteria

To discuss the soundness behavior we tend to might attend the two solvability conditions (64) and (69) and brushing them so the amplitude equation is wanted in terms of the initial variable θ . This might be achieved by adding the multiplication of (64) with and (69) with then setting $\delta \rightarrow 1$, the subsequent nonlinear equation is obtained:

$$\frac{d^2\gamma}{d\theta^2} - i \frac{D'}{D''} \frac{d\gamma}{d\theta} = -\frac{1}{D''} \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} - 3\omega_{12} \right) \gamma^2 \bar{\gamma} - \frac{1}{D''} \left(\frac{3\omega_{12}^2}{D_3} - 10\omega_{23} \right) \gamma^3 \bar{\gamma}^2. \quad (78)$$

This is the cubic–quintic nonlinear Duffing equation having the imaginary damping term and without the natural frequency. In order to perform the oscillatory solution, it is useful for introducing the absent of the auxiliary term [33]. Therefore, equation (72) may be rewritten as

$$\frac{d^2\gamma}{d\theta^2} + \sigma^2\gamma = \sigma^2\gamma + i \frac{D'}{D''} \frac{d\gamma}{d\theta} - \frac{1}{D''} \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} - 3\omega_{12} \right) \gamma^2 \bar{\gamma} - \frac{1}{D''} \left(\frac{3\omega_{12}^2}{D_3} - 10\omega_{23} \right) \gamma^3 \bar{\gamma}^2, \quad (79)$$

where σ^2 is an unknown constant determined from the condition of the uniform solution. The use of the homotopy perturbation technique with a new homotopy parameter can be used to achieve the following condition of the uniform solution [36]:

$$D''\sigma^2 - D'\sigma - 3 \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} - 3\omega_{12} \right) A^2 - 10 \left(\frac{3\omega_{12}^2}{D_3} - 10\omega_{23} \right) A^4 = 0, \quad (80)$$

where A representing a constant, the amplitude of the oscillatory solution of equation (79). Since the stability constraint requires that the parameter σ must be the real quantity which needs that the discriminant of the above solvability condition equation is positive. That is

$$D'^2 + 12D'' \left(\frac{2\omega_{11}^2}{D_2} - 3\omega_{21} - 3\omega_{12} \right) A^2 + 40D'' \left(\frac{3\omega_{12}^2}{D_3} - 10\omega_{23} \right) A^4 > 0. \quad (81)$$

This condition can be satisfied for all values of A^2 and the stability arises when

$$10D'^2D''\left(\frac{3\omega_{12}^2}{D_3}-10\omega_{23}\right)-9D''^2\left(\frac{2\omega_{11}^2}{D_2}-3\omega_{21}-3\omega_{12}\right)^2 > 0. \quad (82)$$

This condition will be investigated numerically in the following sub-section:

5.5 Numerical Estimation for Stability Configuration

In this section, the results of numerical calculation on the stabilization of surface waves propagating through an associate interface between two superposed rotation fluids are communicated. Firstly, it is helpful to research the numerical assess for the linear stability of the wave propagating on the interface. So as to gift this examination, numerical calculations for stability condition (45) are created for each vertical and tangential magnetic field influence. The results of the calculations are displayed in Figs (7) through (11).

The stability condition obligatory from the linear stability analysis has been clarified numerically in Fig (7) - Fig (11). In these graphs, the system is taken into account to be statically unstable wherever the stratified density is chosen to be $\rho > 1$. The plane is partitioned off into stable and unstable regions. The stable region is depicted by the image S whereas the unstable region has been labeled by the image U . The stable region has glad the relation (45). Within the calculation given below, all the physical parameters are wanted within the dimensionless kind as outlined higher than. The stability examination is formed by fixing the worth of all the physical parameters apart from one parameter having varied values for comparison.

The action of the streaming having massive stratified velocity ($V > 1$) and given in Fig. (7). This graph represents the stability diagram for a system $\rho = 1.5$, $k_x = 0.5$, $\sigma_T = 5$ and $\sigma_{H\perp} = 2$. The diagram illustrates the variation of the stratified velocity, $V = 1, 1.3, 1.6,$ and 1.9 , on the stability profile. Inspection of the graph reveals that because the stratified fluid speed is inflated, the unstable region is inflated. This shows a destabilizing influence for the rise of the high speed. The destabilizing influence of the Kelvin-

Helmholtz waves have been incontestable earlier by Chandrasekhar [14]. The linear electro-rheological Kelvin-Helmholtz instability was studied by El-Dib and Matoog [39]. They showed that the streaming contains a strictly destabilizing influence.

The investigation of accelerating the stratified fluid velocity once $V < 1$ is displayed in Fig (8) for the identical system of Fig (7). The diagram shows the variation of the stratified velocity, $V = 0, .2, .4,$ and $.6$, on the stability profile. As seen from the graph, the rise of the low stratified velocity V ends up in shifting the transition curve caused the increase within the stable region. This shows a helpful influence on the stable configuration.

The influences of each normal and tangential magnetic field are displayed in Figs (9) and (10), severally, for the identical system of Fig (7) for a selected worth of $V = 0.5$. Four consecutive values for the vertical magnetic fields or the tangential field are considered thought of in these graphs. It is apparent from the examination of this graph that the increase of the vertical field of force can increase the unstable region causes a destabilizing role.

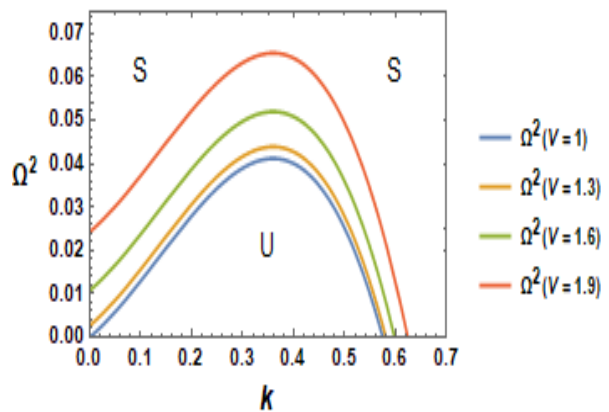


Figure 7: The graph is to illustrate the stability relation (45). The influence of the variation of small-stratified fluid velocity $V < 1$ on the linear stability diagram in the presence of a vertical magnetic field. The system have

$$\rho = 1.5, k_x = 0.5, \sigma_T = 5 \text{ and } \sigma_{H\perp} = 2.$$

Now, the action of the applied vertical magnetic field was switched off, and the contribution of the applied horizontal magnetic field has been displayed in Fig (10) for the same system of Fig (9).

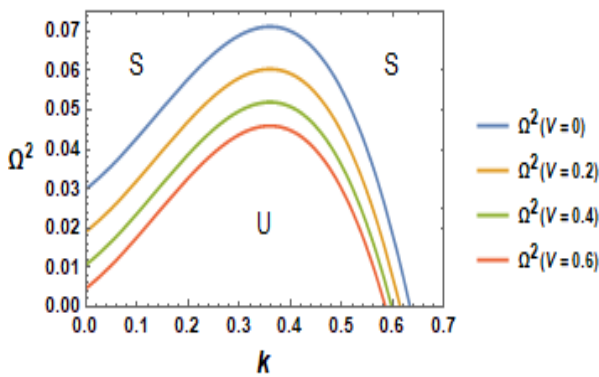


Figure 8: Influence of the variation of the large-stratified fluid velocity ($V > 1$) on the linear stability diagram for the same system of Fig. (7).

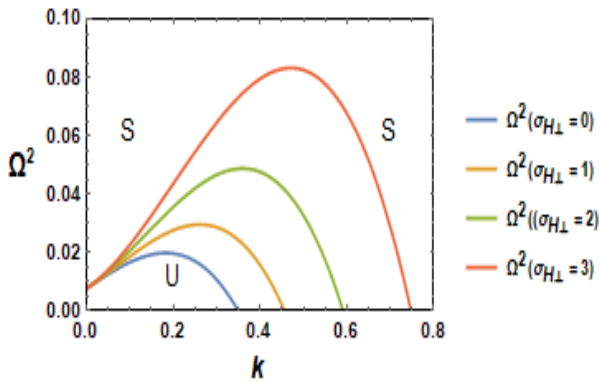


Figure 9: Influence of the variation of the normal magnetic field on the linear stability diagram for the same system of Fig. (1) except that $V = 0.5$.

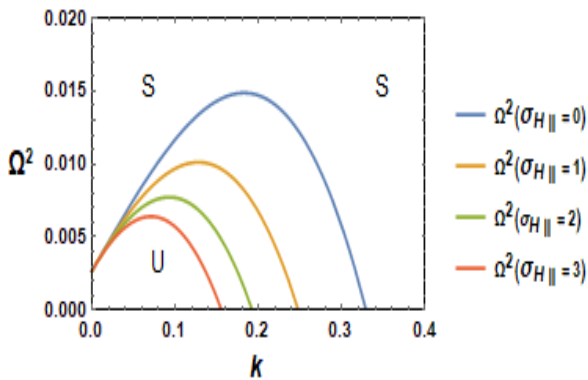


Figure 10: As in Fig. (9) except that the variation of the normal magnetic field is replaced by the tangential field and $\sigma_{H\perp}$ and $\sigma_{H\parallel}$.

The variation of the horizontal magnetic fields is considered as $\sigma_{H\parallel} = 0, 1, 2$ and 3 . It seems that the increase in the values of the horizontal magnetic field plays a stabilizing role. The destabilizing influences of the vertical field and the stabilizing influence of the tangential field

have been demonstrated earlier by Melcher [9] and by other several researchers for inviscid flow through the linear stability theory. Melcher [9] confirmed that in the linear stability theory, the tangential field has a stabilizing action (it increases the surface tension influence), while the normal field has a destabilizing impact (it decreases the surface tension influence).

In Fig (11), the stability picture has been displayed in the plane $(\sigma_{H\perp} - k)$ for variation in the angular velocity $\Omega = 0, 0.5, 1, 1.5, 2$ and 2.5 . In the absence of the angular velocity Ω , the transition curve $\sigma_{H\perp}$ separates the plane into a stable region and an unstable region. It appears that the existence of the angular velocity Ω playing a stabilizing role in which the stable region has increased and the decreased region has decreased in size as the angular velocity parameter is increased. This shows a stabilizing influence on the existence of the angular velocity.

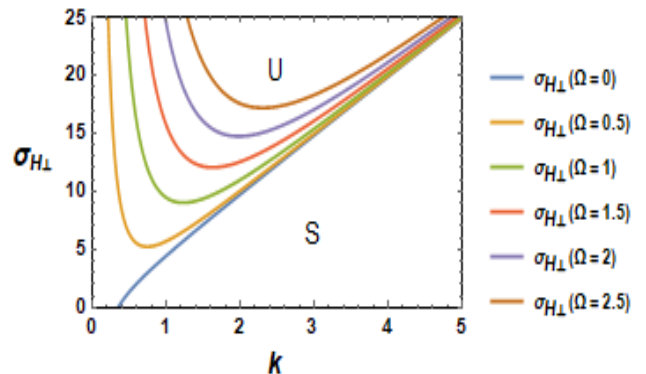


Figure 11: The graph is constructed for $\sigma_{H\perp}$ versus k . Influence of the angular velocity on the linear stability criteria for similar system of Fig. (9).

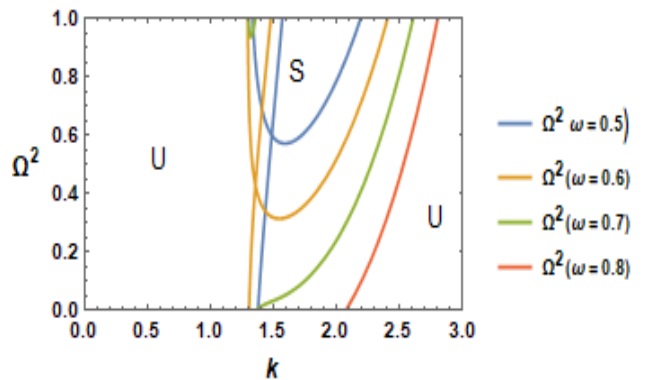


Figure 12: Influence of the variation of the natural frequency ω on the nonlinear stability diagram for the system having $k_x = 1, \rho = 1.5, \sigma_T = 1.2, V = 1.5, \mu = .46$ and $\sigma_{H\perp} = 0.5$.

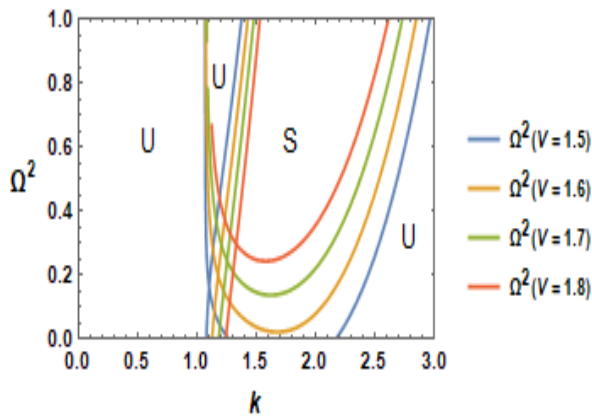


Figure 13: Influence of the variation of large-stratified fluid velocity ($V > 1$) on the nonlinear stability diagram for a similar system as given in Fig. (12) except that $\omega = 1$ and $\sigma_{H\perp} = 1$.

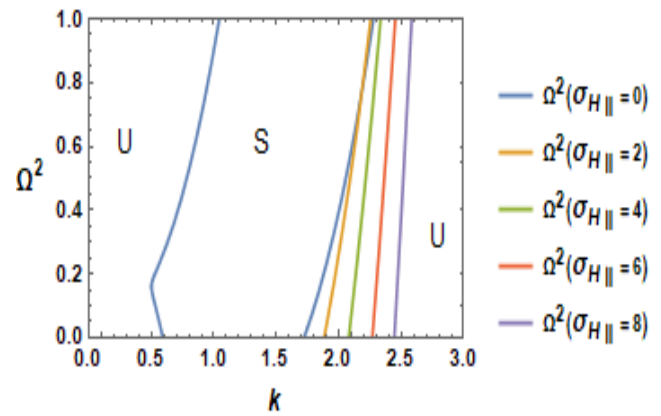


Figure 16: Influence of the variation of the tangential magnetic field for the same system as in Fig (15) except that $k_x = .5$.

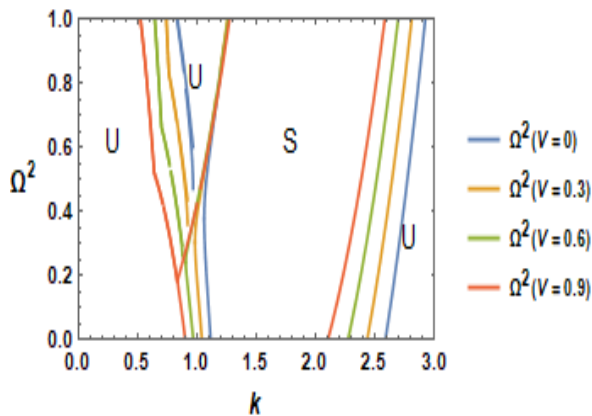


Figure 14: Influence of the variation of low-stratified fluid velocity ($V < 1$) for a similar system as in Fig (13) except that $k_x = 0.5$.

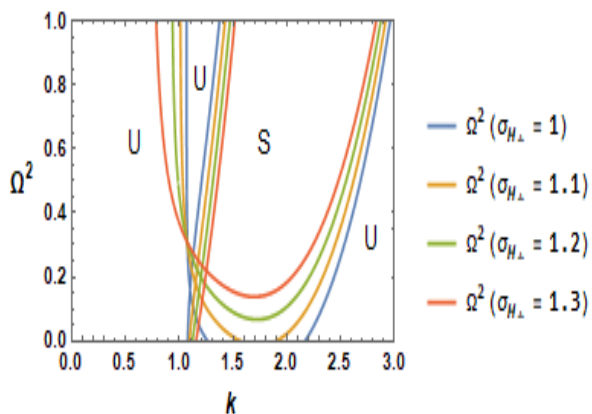


Figure 15: Influence of the variation of the vertical the magnetic field for the same system as in Fig (12) $\omega = 1$ and $V = 1.5$.

In graphing the nonlinear stability image, we have a tendency to present the results of the stability analysis for surface wave propagation through the interface between two streaming rotation fluids. The stability condition (82) that obligatory from the nonlinear analysis has been illustrated within the stability diagram as shown in the graphs as displayed in Figs (12-16). A numerical search was conducted to hunt sequent values for every parameter displayed in these graphs for comparison. The stable region concerned in these graphs was set by satisfying the difference (82).

The careful numerical outcome show that the computed worth for the variation of the natural fluid frequency ω on the stable diagram of the plane $(\Omega^2 - k)$ is used in Fig (12). The system thought-about during this graph is statically unstable. The stable region is because of the action of the rotation parameter. This stable region has increased because of the fluid, the natural frequency ω is increased. This shows the helpful influence for the frequency ω .

The examination of the influence of the variation of the stratified velocity is illustrated in Figs. (13) and (14). Examination of the stability graph reveals that the increase in the large stratified velocity ($V > 1$), as well as the increasing in the low stratified velocity ($V < 1$), leads to a decrease in the stable region. This shows the destabilizing influence of the stratified velocity in the nonlinear stability graph. The same rule is observed in the linear stability examination for

the variation of the large stratified velocity, while there is a contrast to the influence in the variation of low stratified velocity.

When the transient curves are plotted as a function in the vertical magnetic field or as a function of the horizontal magnetic field the results are displayed in Fig (15) and Fig (16), respectively. The examination of the impact of the variation in the vertical magnetic field and the influence of the horizontal magnetic field on the nonlinear stable diagram has the same role as in the linear stability graph as illustrated in Fig (9) and Fig (10), respectively.

The examination of the influence of the alteration of the stratified velocity is illustrated in Figs. (13) and (14). Examination of the stability diagram reveals that the rise within the massive stratified rate ($v > 1$), yet because the increasing within the low stratified rate ($v < 1$), ends up in a decrease within the stable region. This shows the destabilizing influence for the stratified rate within the nonlinear stability diagram. The identical role is ascertained within the linear stability examination for the variation of the massive stratified rate, whereas there's a distinction for the influence within the variation of the low stratified rate. Once the transient curves, square measure premeditated as a performance within the vertical field of force or as a performance of the tangential field of force the results square measure displayed in Fig (15) and Fig (16), severally. The examination of the impact of the alteration within the normal field of force and also the action of the horizontal field of force on the nonlinear stable graph has an identical role as in the linear stability graph as illustrated in Fig (9) and Fig (10), severally.

6 Conclusion

We have investigated the impact of rotating on the flow and surface pattern formation in three-dimension nonlinear Kelvin-Helmholtz instability. The system is stressed by a vertical or a horizontal direction of the field in the separation face of two rotating semi-infinite same and incompressible fluids. Capillary-gravity waves of the permanent kind at the interface between two magnetic fluids in a very rotating frame of reference have an interest. Allowance

low rotation is performed. The solutions of the equations of motion under nonlinear boundary conditions cause etymologizing an equation that governs the surface displacement having high nonlinearity. He-multiple-scales technique is employed to expand the governing high nonlinear equation. The system is delineated by a generalized homotopy equation. This equation is accomplished by utilizing the cuboidal quintic nonlinearity. Taylor theory is employed to expand the nonlinear dispersion relation. Additionally, the perturbation analysis results in imposing two levels of the solvability conditions, that square measure accustomed construct the cubic-quintic nonlinear Duffing equation. The stability criterion square measure derived from finding the Duffing equation. Moreover, the approximate answer of the nonlinear dispersion relation is performed in terms of the surface displacement. Each the stability criteria and therefore the surface displacement answer square measure illustrated diagrammatically. Numerical calculations showed that the presence of low rotation to the Kelvin-Helmholtz drawback can suppress the instability due to the streaming of the flow. The vertical field is enjoying an unstable role, whereas the existence of the tangential field plays a stabilizing influence.

7 Competing Interests

The authors declared that they do not have any conflict of interest in the publication of this paper.

How to Cite this Article:

Y. El-Dib and A. Mady, "He's Multiple-Scale Solution for the Three-dimensional Nonlinear KH Instability of Rotating Magnetic Fluids", *Int. Ann. Sci.*, vol. 9, no. 1, pp. 52-69, Dec. 2019. doi:10.21467/ias.9.1.52-69

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