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SUPPLEMENTARY FILE

Appendix A: Derivations

A.1 $\pi/2$ pulse as an eigenbasis transformation

We want a transformation U from the $\{|+_{L,R}\rangle, |-_{L,R}\rangle\}$ basis to the $\{|\pm 1, \mp 1\rangle, |\pm 3, \mp 3\rangle\}$ basis. Let $\{|+_{L,R}\rangle, |-_{L,R}\rangle\} \equiv \{a_i\}$ and $\{|\pm 1, \mp 1\rangle, |\pm 3, \mp 3\rangle\} \equiv \{b_i\}$. Then for a one-to-one mapping we need a matrix U such that

$$|b_i\rangle = U|a_i\rangle$$

Multiplying $\langle a_j|$ to both sides,

$$\langle a_j|b_i\rangle = \langle a_j|U|a_i\rangle = U_{ij}$$

Working in the vector space of $\{|\pm 1, \mp 1\rangle, |\pm 3, \mp 3\rangle\}$ this gives

$$\begin{cases} U_{12} = \langle -_{L,R}|\pm 1, \mp 1\rangle = \frac{1}{\sqrt{2}}[1 \quad -1]\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}} \\ U_{12} = \langle -_{L,R}|\pm 1, \mp 1\rangle = \frac{1}{\sqrt{2}}[1 \quad -1]\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}} \\ U_{21} = \langle +_{L,R}|\pm 3, \mp 3\rangle = \frac{1}{\sqrt{2}}[1 \quad 1]\begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{\sqrt{2}} \\ U_{22} = \langle -_{L,R}|\pm 3, \mp 3\rangle = \frac{1}{\sqrt{2}}[1 \quad -1]\begin{bmatrix}0\\1\end{bmatrix} = -\frac{1}{\sqrt{2}} \end{cases}$$

So a transformation matrix is given by

$$U = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}$$

A $\pi/2$ pulse is given[3] by the matrix

$$\frac{\hat{\pi}}{2} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\-i & 1\end{bmatrix}$$

Which has similar form to the generic phase-preserving transformation and maps according to

$$\begin{cases} \frac{\hat{\pi}}{2}|+_{L,R}\rangle = \frac{1}{\sqrt{2}}(1+i)|\pm 1, \mp 1\rangle + \frac{1}{\sqrt{2}}(1-i)|\pm 3, \mp 3\rangle i \quad \rightarrow \quad P(|\pm 3, \mp 3\rangle) = \left|\frac{1}{\sqrt{2}}(1+i)\right|^2 = 1 \\ \frac{\hat{\pi}}{2}|-_{L,R}\rangle = \frac{1}{\sqrt{2}}(1-i)|\pm 1, \mp 1\rangle + \frac{1}{\sqrt{2}}(-1-i)|\pm 3, \mp 3\rangle i \quad \rightarrow \quad P(|\pm 3, \mp 3\rangle) = \left|\frac{1}{\sqrt{2}}(-1-i)\right|^2 = 1 \end{cases}$$

This accomplishes the same up to an arbitrary phase, which does not matter because we immediately measure the state in a phase-independent way.

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Appendix B: Calculations

B.1 Magnetic Field components

Here we use Mathematica to calculate the resulting magnetic fields at the trap place by using Eq. (23) and grouping it by components. We use a symmetric equation for the x-oriented coil except that it is opposite positionally of where the z-axis coil is with respect to its axis, so $\hat{x}$ goes to $-\hat{x}$, as well as Eq. (24).

```
In[44]:= ntx == 31; nlx = 7; nty = 7; nly = 7; ntz = 31; nlz = 7;

In[40]:= Bz == Sum[Sum[((4*Pi*10^-7) * Iz * (j * .00091 + .061)^2 *
            (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2))) /
          (2 * ((j * .00091 + .061)^2 + ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2))^(3/2)) *
         (1 + (15 * (j * .00091 + .061)^2 * (0.0965 + i * 00.91 * 10^-3)^2 *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2) /
            (4 ((j * .00091 + .061)^2 + .0555^2 + (0.0965 + i * 00.91 * 10^-3)^2)^2)) *
         (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2)) +
        ((4 * Pi * 10^-7) * Iz * (j * .00091 + .061)^2 * 5.56 / (√(5.56^2 + (7.83 + i * .091)^2))) /
          (4 * ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^(5/2)) *
         (2 * (j * .00091 + .061)^2 - ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) +
           (15 * (j * .00091 + .061)^2 * ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2 *
              (4 * (j * .00091 + .061)^2 - 3 * (0.0965 + i * 00.91 * 10^-3)^2)) /
            (8 ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^2)) *
         5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)), {i, 0, ntz}], {j, 0, nlz}]

Out[40]= Bz == 0.000189744 Iz

In[41]:= Bx == Sum[Sum[-((4 * Pi * 10^-7) * Ix * (j * .00091 + .061)^2 *
             (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2))) /
          (2 * ((j * .00091 + .061)^2 + ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2))^(3/2)) *
         (1 + (15 * (j * .00091 + .061)^2 * (0.0965 + i * 00.91 * 10^-3)^2 *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2) /
            (4 ((j * .00091 + .061)^2 + .0555^2 + (0.0965 + i * 00.91 * 10^-3)^2)^2)) *
         (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2)) -
        ((4 * Pi * 10^-7) * Ix * (j * .00091 + .061)^2 * 5.56 / (√(5.56^2 + (7.83 + i * .091)^2))) /
          (4 * ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^(5/2)) *
         (2 * (j * .00091 + .061)^2 - ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) +
           (15 * (j * .00091 + .061)^2 * ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2 *
              (4 * (j * .00091 + .061)^2 - 3 * (0.0965 + i * 00.91 * 10^-3)^2)) /
            (8 ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^2)) *
         5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)), {i, 0, ntx}], {j, 0, nlx}]

Out[41]= Bx == -0.000189744 Ix
```

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In[42]:= By == Sum[Sum[((4 * Pi * 10^-7) * Iz * (j * .00091 + .061)^2 *
            (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2))) /
          (2 * ((j * .00091 + .061)^2 + ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2))^(3/2)) *
         (1 + (15 * (j * .00091 + .061)^2 * (0.0965 + i * 00.91 * 10^-3)^2 *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2) /
            (4 ((j * .00091 + .061)^2 + .0555^2 + (0.0965 + i * 00.91 * 10^-3)^2)^2)) *
         .0555 / (√(5.56^2 + (7.83 + i * 0.091)^2)) -
        ((4 * Pi * 10^-7) * Iz * (j * .00091 + .061)^2 * 5.56 / (√(5.56^2 + (7.83 + i * .091)^2))) /
          (4 * ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^(5/2)) *
         (2 * (j * .00091 + .061)^2 - ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) +
           (15 * (j * .00091 + .061)^2 * ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2 *

              (4 * (j * .00091 + .061)^2 - 3 * (0.0965 + i * 00.91 * 10^-3)^2)) /
            (8 ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^2)) *
         (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2)), {i, 0, ntx}],
      {j, 0, nlx}] + Sum[Sum[((4 * Pi * 10^-7) * Ix * (j * .00091 + .061)^2 *
            (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2))) /
          (2 * ((j * .00091 + .061)^2 + ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2))^(3/2)) *
         (1 + (15 * (j * .00091 + .061)^2 * (0.0965 + i * 00.91 * 10^-3)^2 *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2) /
            (4 ((j * .00091 + .061)^2 + .0555^2 + (0.0965 + i * 00.91 * 10^-3)^2)^2)) *
         .0555 / (√(5.56^2 + (7.83 + i * 0.091)^2)) -
        ((4 * Pi * 10^-7) * Ix * (j * .00091 + .061)^2 * 5.56 / (√(5.56^2 + (7.83 + i * .091)^2))) /
          (4 * ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^(5/2)) *
         (2 * (j * .00091 + .061)^2 - ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) +
           (15 * (j * .00091 + .061)^2 * ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2) *
              (5.56 / (√(5.56^2 + (7.83 + i * 0.091)^2)))^2 *
              (4 * (j * .00091 + .061)^2 - 3 * (0.0965 + i * 00.91 * 10^-3)^2)) /
            (8 ((j * .00091 + .061)^2 + (0.0965 + i * 00.91 * 10^-3)^2 + .0555^2)^2)) *
         (7.8297 + i * 0.091) / (√(5.56^2 + (7.83 + i * 0.091)^2)), {i, 0, ntz}],
      {j, 0, nlz}] - Sum[Sum[((4 * Pi * 10^-7) * Iy * (j * .00091 + .061)^2) /
        (2 * ((j * .00091 + .061)^2 + ((0.0965 + i * 00.91 * 10^-3)^2 + .0555^2))^(3/2)), {i,
        0, nty}], {j, 0, nly}]
Out[42]= By == 0.000024984 Ix - 0.000247402 Iy + 0.000024984 Iz
```